UNIVERSITY OF LIVERPOOL

GDP-related Mortality Model for Insurance Pricing and Longevity Securitisation

Thesis submitted in accordance with the requirements of the University of Liverpool for the degree of Doctor in Philosophy by

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"Useful knowledge when it is useful to others."

Al-Quantani

Abstract

Since the beginning of 20th century, human longevity has been increasing tremendously. The growth of ageing population is evidence for positive milestones of a society's well-being. Apparently, socio-economic status is commonly conceptualised as a social standing of an individual or society. Higher socio-economic status has long been identified as a contributing factor for mortality improvement. The ongoing increases of ageing population have alarmed many organisations especially pension fund providers and insurance companies offering annuities and life insurances. Furthermore, the evolution of longevity is believed to produce a new curve on the standard life expectancy as its shift has underestimated the future life expectancy. The purpose of this study is to develop a robust mortality model factoring the macroeconomic element. It is motivated by the evolution of longevity that causes insufficient to pension and annuities funding, inaccurate estimation of life tables, under-pricing of financial products and under-estimation of longevity risk.

First, this study extents the works from previous mortality models, in particular O'Hare-Li (OL) model, by considering the influencing factor of Gross Domestic Product (GDP) into the new model. Hence, the development of the new model (OL-GDP) is through the combination of OL model and the GDP-age dependent factor. The OL-GDP model is then applied to the selected Eurozone countries. The results show that central mortality rates estimated by OL-GDP are robust and have better fitting relative to alternative models.

Second, this study analyses the robustness of the OL-GDP model given various changes in its parameters. Sensitivity analyses are conducted to study the impact of the model towards various modifications of each parameter. The study is then extended on analysing its impact on insurance pricing and reserving through the calculations of the Actuarial Present Values (APVs) of life insurance and annuity products. Undoubtedly, OL-GDP model maintains its robustness given changes in various parameters on most actuarial products as compared to other mortality models under study.

Thirdly, as for risk management measures, this study demonstrates the approach of natural hedging between two portfolios, annuity and life insurance. This approach is more suitable for a company that underwrites both businesses, life and

annuity. The management of the liability outflows can be done internally within the company. Evidently, the application of the natural hedging within a company, provides stability of the company's outflows and better management of its insurance liabilities. In addition, this study introduces an alternative risk transfer tool for managing longevity by issuing longevity bond. Hence, this study proposes methods of structuring and pricing a longevity bonds as an alternative risk transfer tool for managing longevity risk.

The contribution to the academic literature is threefold. On the theoretical side, building on the work of existing mortality models, this study proves that macroeconomic factors have significant influence on mortality models. On the empirical side, the OL-GDP model has higher sensitivity towards insurance pricing and reserving as compared to alternative models. Moreover, the OL-GDP model maintains its robustness to variations within the parameters of the model.

Generally, the OL-GDP model successfully reproduces better mortality estimations for a significant number of Eurozone countries. Moreover, OL-GDP provides more plausible forecasts that may assist insurers and pension providers to improve their insurance pricing and reserving strategies. Finally, due to its robustness, this model is reliable to help in improving a company's risk management framework in managing its insurance liabilities of life and annuity portfolios efficiently. What is more, as an alternative risk transfer tool, this study proposes methods for structuring and pricing the longevity bond.

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Contents

List of Publications and Presentations

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7. *Implementing Temperature and Lifestyle Risk Factors on Mortality Models : Commonwealth Countries*. In the 2nd International Conference on Statistics in Science, Business and Engineering 2015 (ICSSBE2015). September 2015. Putrajaya, Malaysia.

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Abbreviations

AIC	**A**kaike **I**nformation **C**riterion
APV	**A**ctuarial **P**resent **V**alue
ARIMA	**A**uto **R**egressive **I**ntegrated **M**oving **A**verage
BIC	**B**ayesian **I**nformation **C**riterion
CSS	**C**ubic **S**moothing **S**pline
ETS	**E**xponential **S**moothing
FV	**F**uture **V**alue
GDP	**G**ross **D**omestic **P**roduct
HMD	**H**uman **M**ortality **D**atabase
KPSS	**K**witatkowski **P**hillips **S**chmidt **S**hin
LC	**L**ee **C**arter
LC-GDP	**L**ee **C**arter **-** **G**ross **D**omestic **P**roduct
MAD	**M**ean **A**bsolute **D**eviation
MAPE	**M**ean **A**bsolute **P**ercent **E**rror
MPE	**M**ean **P**ercentage **E**rror
OL	**O**'Hare **L**i
OL-GDP	**O**'Hare **L**i **-** **G**ross **D**omestic **P**roduct
PV	**P**resent **V**alue
PVFB	**P**resent **V**alue **F**uture **B**enefit
PVFB	**P**resent **V**alue **F**uture **P**remium
S&P	**S**tandard & **P**oor's

To my husband, mother, late father, parents in-law,
daughters and son

Chapter 1

Introduction

The evolution of human longevity has been made significant ever since the beginning of the 20th century. Many industrialised countries witnessed a consistent rising of human longevity at a promising growth. Factors like socio-economic status, medical innovations, nutrition habits and environmental conditions are believed to be the contributing aspects (French and O'Hare, 2014; Gaille and Sherris, 2010). Each country experiences a different trend of longevity improvement. For instance, the life expectancy for European countries has risen significantly over the past few decades. This has led insurance and pension providers to revisit their way to price their products and to hedge the longevity risk. The key to this exercise is to have a robust mortality model.

1.1 Background of the Study

Life expectancy is one of the most important indicators used to compare different population groups in demography studies. Statistically, it is defined as the mean number of years a cohort of people might expect to live based on the year of birth, current age and other factors including sex. The ongoing increases of life expectancy followed by the significant increase of ageing population have alarmed many organisations especially pension providers such as governments, pension funds, and insurance companies offering annuities and life insurances (Bloom et al., 2007; Hari et al., 2008; Pitacco et al., 2009). It is believed that the evolution of this longevity trend has produced new curves to the standard life expectancies, making the current life tables underestimate the future life expectancy.

The government faces challenges on financing older people with longer life spans whilst insurers are liable for providing conducive and appropriate health care solutions sufficiently. In response to these concerns, the issue of longevity risks must be well addressed by the related bodies. This is to ensure that governments fund the pension system sufficiently, and actuaries and insurers price their age-related financial instruments appropriately.

In a similar way, the premise of this thesis is about the issue of longevity risk. Due to the under-estimation of current life tables, this study developed a new mortality model for the selected Eurozone countries. As the potential reasons for mortality improvements comprise of socio-economic status, technological advancement in medicine, nutrition and diet, lifestyle, and living environment (Gaille and Sherris, 2010; Roy, 2012; French and O'Hare, 2014), this study utilises the factor of economics impact, GDP, in modelling the new mortality model. The model, OL–GDP (extended model of O'Hare and Li (2012)), is then being used for all other studies in this thesis. The robustness of the model is compared against other mortality models namely Lee and Carter (1992) model (LC), O'Hare and Li (2012) model (OL) and Niu and Melenberg (2014) model (LC-GDP), as this model incorporates the GDP factor into the LC model.

Subsequently, this study analyses the model's stability by observing variations within the parameters of the model. Variations in age range, correlation coefficients, age parameter, time dependent factor k_t^2 and forecasting methods are among the factors analysed in this study. In addition, this study investigates the impact of insurance pricing and reserving in respect to related various changes within the Actuarial Present Values (APVs) calculations. This includes variations of mortality models, forecasting methods and interest rates.

Nevertheless, in mitigating the issue of longevity risk, this study undertakes the concept of natural hedging that applies within a company. This internal approach optimises the allocation of annuities and term insurance corresponding to mortality improvement and deterioration. Moreover, this study proposes the next step of hedging the longevity risk by issuing a longevity bond. The framework of this approach is based on Lorson and Wagner (2014), where the most important parts in pricing the bond are defining the tranche and excess loss process, and the computations of principal payment and coupon payment.

1.2 Problem Statement

Considered as one of the positive drives in raising the society's well-being, socio-economic status is said to have a significant impact on the mortality improvements for some countries. Factors conducive to mortality improvement vary for each country, thus, each country experiences a different trend of mortality improvement (Roy, 2012). For instance, the life expectancy for European countries has risen significantly over the past few decades. This has led the insurance and pension providers to revisit the way their price the products and hedge the longevity risk. The key to this exercise is to have a robust mortality model.

The evolution of longevity is believed to produce a new curve on the standard life expectancy. Apparently, this shift has underestimated the current one. Therefore, an accurate measure of estimating the future life expectancy is deemed crucial for holders of longevity risks like pension funds, life insurers and annuity providers. They need to be more mindful of funding, pricing and reserving their products sufficiently. On another development, evidence of macroeconomic impact on mortality has been presented by many researchers (Preston, 2007; Granados, 2008; Hanewald, 2011; Niu and Melenberg, 2014; Rolden et al., 2014; Boonen and Li, 2017). These findings lead to a new endeavour of exploring this relationship. This study takes a step forward on improving the existing mortality rates by considering the influence of a macroeconomic factor, namely GDP.

Knowing that some important parameters in the mortality model are sensitive to the direction of future mortality rates, this study analyses the reactions of the mortality rate estimations given variations in the model's parameters. Furthermore, even small changes in mortality rates may have an impact on the age-related financial instruments like annuity and life insurances. Thus, a comprehensive study on how to arrive at the most optimum price that ensures sufficiency is essential. For this reason, this study provides a number of quantifying ways on the impact of these variations on various financial calculations.

As part of the risk management framework in mitigating longevity risk, various issues pertaining to pricing and reserving the insurance and annuity products have increasingly become major challenges. Natural hedging is considered as a self-managed risk, executed internally within the company. This execution is more appropriate for a company that writes both life insurance and annuity businesses. Natural hedging benefits the company through stabilising its liabilities outflow.

Companies can reduce their liabilities if both businesses are being transacted concurrently. Nevertheless, striking a balanced business portfolio is difficult because of the demand and supply rule. The portfolio between life insurance and annuity may not always be balanced. Therefore, as part of an alternative risk transfer tool, transferring risks to financial markets is seen to be efficient in pooling the risk for hedging longevity successfully. Structuring and pricing longevity risk using modern securitisation methods, has become a heuristic and provided avenues for researchers to explore further. This study considers the securitisation of longevity risk focusing on structuring and pricing a longevity bond using techniques developed by Lorson and Wagner (2014) and Kim and Choi (2011). A model based on Eurozone countries mortality data and calibrated to insurance risk linked market data is used to assess the structure and market consistent pricing of a longevity bond.

1.3 Research Aims and Objectives

This study aims to develop a robust mortality model considering the influential macroeconomic factor of gross domestic product (GDP). The research objectives are as follows:

1. To find the best model fitting the mortality rates for the selected Eurozone countries;
2. To analyse the sensitivity of the insurance pricing and reserving given changes on the model parameters; and
3. To examine the impact of mortality risk and propose a framework of structuring longevity bonds as part of mortality risk management.

1.4 Significance of the Study

Despite the growing interest in mortality and longevity, there are few empirical studies that examine the application of these risks on the insurance pricing and risk management framework. Having said that, numerous attempts have been applied to model mortality experience. However, the application of GDP in mortality rate is considerably new. Thus, this study offers two-prong evidences. On the theoretical side, factoring GDP helps to improve the robustness of existing mortality models. On the other hand, new empirical evidence suggests that the GDP-related model results in better insurance pricing as compared to other models and the longevity risk can be hedged with securitisation.

Evidently, with the inclusion of GDP growth indicator in the OL model, this study finds the best fitting model for most Eurozone countries, especially for France, Germany and Italy. Except for female, male and unisex produced the best fitting of OL-GDP as compared to other models. This study notes the impact of GDP is significant especially for the most populous countries. What is more, the proposed model also demonstrates better quality forecasts of mortality rates through the applications of ARIMA process and Exponential Smoothing techniques.

Essentially, the Mean Absolute Percentage Error (MAPE) measures of forecasting shows a significant improvement in forecasting results for shorter (5 years) and longer (15 years) period of times in comparison to the LC model which acts as the reference model. Hence, the impact of economic growth on mortality dynamics has proved to be noteworthy. Further investigation on this will certainly give some light on mortality studies. An in-depth study on this will absolutely give clearer direction on this relationship, explain the mortality trends precisely, and at the same time improve the mortality forecasting accuracy.

In addition, this study considers the impact of insurance and annuity pricing on the GDP-related mortality model. Reflecting the variations and range of modifiable factors involved in the model, the analysis shows that generally, OL-GDP model generates better results relative to other models on the actuarial present values (APVs) calculations. APVs are typical calculations for obtaining appropriate amounts for benefit-payment or series of payments associated with life insurance and life annuities. Factors like choice of data period and forecasting methods, also variations of correlation coefficients, interest rates, age groups and age parameters have been considered in assessing their impact on pricing the insurance products. Sufficient reserves are crucial for all regulated insurers as they are required to keep aside reserves for managing future liabilities. Reserves play an important role in assessing the financial condition of an insurer. Reserves are also important in assessing the solvency of an insurer, in terms of its ability to meet its liabilities.

Moreover, reserves are also important in pricing the insurance products more accurately. Actuaries price the insurance products by estimating the future cost of claims on risks yet to be paid off to the insured by extrapolating the past paid and reserved claim cost.

Furthermore, this study contributes to the current discussion about how insurers can hedge the longevity risk naturally and internally within their insurance and annuity portfolios. Natural hedging aims to stabilise the liabilities outflow of the insurer. Insurer may reduce its mortality risks if both insurance and annuity business are transacted concurrently. On the other hand, as an alternative to natural hedging, this study has introduced another approach of hedging the longevity risk. The longevity risk can also be transferred to other platforms like financial markets. The act of transferring the risks of the porfolio to the third party is called securitisation. Following this, this study has introduced longevity bonds. The framework on how this bond is being structured and priced, is being discussed accordingly.

1.5 Research Questions

The mortality rates are not static and they have been influenced by various factors like lifestyle, biophysical, environment and macroeconomic factors. The latter have the most impact in socio-economic terms. Therefore, macroeconomic factors like the gross domestic product (GDP) have the greatest influence since they are also closely related to other factors such lifestyle, physical well-being and the surrounding environment.

With that, this study would like to understand the level of influence that GDP has on mortality rates. Thereafter, understanding the impact that the adjusted mortality rate has on insurance pricing and reserving as well as measuring mortality risk in respect to improvement and deterioration in mortality rates. Moreover, how the insurers and pension providers manage insurance liabilities given changes in mortality rates in the future. The above can be answered by addressing the research questions in achieving the thesis' aims and objectives.

1. How does the new model improve the estimation of central mortality rates and its forecasting power?;

2. How does the new model respond to variations in parameters for sensitivity of insurance pricing, reserving and insurance liabilities?; and

3. How to hedge the mortality risk such as longevity risk for life insurance and annuity portfolios?.

1.6 Research Design

In meeting those research objectives, this study conducts quantitative time-series analysis. First, model estimation and fitting as well as testing. Second, model testing for the insurance pricing sensitivity. Third, structuring and pricing the longevity risk with securitisation.

More specifically, this thesis is structured as follows. First, this study focuses on the nine most populated Eurozone[1] countries and analyses the effect of socio-economic factors on mortality trends. Namely, this study considers Austria, Belgium, Germany, Greece, France, Italy, the Netherlands, Portugal and Spain. This study grouped these countries into two categories, the strong economic countries like Austria, Belgium, Germany, France and the Netherlands (these countries are long known to hold strongest economic position nationally, while keeping their momentum high by joining Eurozone); and the crisis countries like Greece, Italy, Portugal and Spain (these countries suffered economically and were unable to repay or refinance their government debts following the global crisis in 2008).

Second, this study examines the impact of the new model on its ability to forecast the mortality rates if the parameters involved in the model are modified. Further analysis would also be conducted to study each parameter's sensitivity against the model. Subsequently, the next focus of this study is to demonstrate each parameter's impact on pricing and reserving the annuity and insurance products. The

[1] The Eurozone was established on 1 January 1999 with its first members of 11 countries. It was then enlarged to 19 countries namely Austria, Belgium, Cyprus, Estonia, Finland, France, Germany, Greece, Ireland, Italy, Latvia, Lithuania, Luxembourg, Malta, the Netherlands, Portugal, Slovakia, Slovenia and Spain. These 19 countries are out of the 28 European Union (EU) member states that are grouped together within the same monetary union, using the Euro (€) as their common currency and sole legal tender. The monetary authority of the Eurozone is called Eurosystem.

new model allows for simple quantitative measures of those parameters in connections to mortality forecasts. This is crucial in developing long-term mortality projections, which an important issue in life insurance and pensions.

Third, this study introduces a model for annuity securitisation with the help of longevity bond. At the same time, this study has taken the perspective of the issuing insurer and calculated the price of hedging for the company. This study applied a tranching approach for the securitisation based on the percentile tranching method and designed the securities in a way that the principal payments are risky, i.e. depend on the survival rates of the underlying portfolio of annuitants. To do so, this study first used the OL-GDP model for European countries and calculated estimates on future mortality rates. Next, the tranching process is grouping the portfolio into different rated tranches according to different risk profiles as suggested by S&P ratings for insurance-linked securities. Finally, this study determined the price of the longevity bond for hedging the contracts against longevity risk.

1.7 Overview of the Thesis

The following chapters in this thesis are structured as follows. Chapter 2 explains the macroeconomic factor of GDP that influences mortality rates and the establishment of a new GDP-related mortality model. Thereafter, based on the new GDP-related mortality model, insurance pricing and reserving impact are analysed in Chapter 3. Subsequently, the new GDP-related mortality model is further applied in Chapter 4 for examining the mortality risk and structuring longevity securities. Chapter 5 concludes this thesis.

In Chapter 2, an introduction starts in Section 2.1, expectation, explanation and extrapolation of life expectancy are discussed in Section 2.2. Whereas, Section 2.3 briefly reviews the main mortality models and this study further investigates the relationship between economic growth (GDP) and mortality rates. Section 2.4 focuses on the statistical analyses of the data for the nine Eurozone countries (in our study, Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain), derived from the Human Mortality Database (HMD), followed by a descriptive analyses of the long run relationship among time series data using the Johansen (1988) co-integration test, and other various correlation tests. Based on the findings and comparison studies, this study proposes a new GDP-related model in Section 2.5, with detailed discussion of the fitting process. The forecasting performance using ARIMA process and Exponential Smoothing is discussed in Section 2.5. Finally, Section 2.6 concludes the whole discussion of the study.

Chapter 3 is organised as follows. An introduction is discussed in Section 3.1. Section 3.2 presents an analysis of the impact of various factors involved in the OL-GDP. Based on the outcomes of Section 3.2, the sensitivity analyses are presented in Section 3.3. This section presents the impact of insurance pricing for selected actuarial products given changes of parameters on the OL-GDP model and other mortality models under study.Section 3.4 presents the results of actuarial reserve for selected actuarial products. Finally, Section 3.5 concludes the whole discussion of this chapter.

Chapter 4 is organised as follows. An introduction is discussed in Section 4.1. Section 4.2, demonstrates the idea of natural hedging that can be done internally within the company. Section 4.3 analyses the economics of mortality risk and the needs for mortality securitisation. Section 4.4 proposes the mortality securitisation framework for annuity portfolio by introducing a longevity bond. Section 4.5 concludes.

Lastly, Chapter 5 concludes this study by showing the robustness of the OL-GDP model together with empirical testings. In addition, Chapter 5 presents the limitations of the work done and proposed directions for future research. Finally, this study recommends areas for applying and extending the OL-GDP model in future studies.

1.8 Conclusion

A change in mortality rate experience has become a serious concern for insurers and pension providers. Hence, better life expectancy has led to underestimation of future life expectancy. Consequently, this has caused deficits in pension funds for instance. Having a robust mortality robust helps insurers and pension providers manage their mortality risk more prudently. However, the challenge of modifying mortality projections is made more interesting since the mortality improvements are not comparable across all countries. Some studies have found that different countries experience different stages of demographic transition due to different patterns of population growth. This scenario can be clearly observed when comparing developed and emerging countries. Thus,an accurate identification of factors affecting the mortality experience in a particular country is crucial. On the other hand, evidence of a relationship between economic changes and mortality in many developed and developing countries has been presented by many researchers.

Against this background, this study aims to develop a new mortality model that considers the influences of macroeconomic factors like growth in gross domestic product (GDP) for the nine most populated Eurozone countries and analyse the effect of socio-economic factors on mortality trends. The selected countries are Austria, Belgium, France, Germany, Greece, Italy, Netherlands, Portugal and Spain. Data from 1960 to2007 (except: Germany: 1970–2007, Greece: 1981–2007). In achieving the goal, this study focuses on three research objectives. First, this study analyses the relationship between GDP and mortality rate as well as its robustness in the nine countries. This study validates the proposed model with several approaches like MAPE, MAD and BIC analysis, and investigates the goodness of fit of all alternative models, i.e. Lee and Carter (LC), O'Hare and Li (OL), LC-GDP and OL-GDP. Second, it further examines the impact of the new mortality towards insurance pricing and reserving. Third, to investigate the insurers and pension providers' cash flow sensitivity given the improvement/deterioration in mortality rate.

In meeting those research objectives, this study conducts a quantitative time-series analysis. First, by improving the existing model with GDP for model regression and fitting as well as statistical testing. In addition the new model is tested with forecasting capability in determining its accuracy relative to the alternative models commonly referred in academic studies. Second, the model is statistically tested for insurance pricing and reserving impact to the insurers and pension providers. Third, conducting sensitivity analysis on improvement or deterioration in mortality rates that leads to structuring and pricing of the longevity risk with securitisation, for instance, longevity bond.

The importance of this study can be three pronged. On the theoretical perspective, this study improves the existing mortality models by considering the influence of macroeconomic factor that best explain the socio-economic trends in selected Eurozone countries. As for the empirical side, this study shows better capabilities of the model in analysing the impact of insurance pricing and reserving relatives as compared to the existing models. Lastly, this study helps insurers and pension providers respond to the change (improvement or deterioration) in mortality rates. The changes have put a great impact on cash flows and liabilities to the insurers and pension providers. Therefore, the key is to have a robust mortality model that works across countries.

Chapter 2

Life Expectancy with OL-GDP Model

Socio-economic status is commonly conceptualised as the social standing or well-being of an individual or society. Higher socio-economic status has long been identified as a contributing factor for mortality improvement. This paper studies the impact of macroeconomic fluctuations with GDP as a proxy on mortality for the nine most populous Eurozone countries. Based on the statistical analysis between time-dependent indicator of Lee and Carter model and GDP, and adaptation of the good features of the O'Hare and Li model, a new mortality model including this additional economic-related factor is proposed. Results for male and female from ages between 0–89, and similar for unisex data are provided. This new model shows a better fitting, and more plausible forecasts among a significant number of Eurozone countries. An in-depth analysis of the findings is provided to give a better understanding of the relationship between mortality and GDP fluctuations.

2.1 Introduction

Since the beginning of the 20th century, human longevity has been increasing tremendously. The growth of ageing population is evidence for a positive milestone of society's well-being. Each country experiences a different trend of longevity improvement (Pampel, 2005; Yang and Wang, 2013). For instance, the life expectancy for European countries has risen significantly over the past few decades as shown in Figure 2.1. Life expectancy rose by 14 years from 1950 to 2015[1].

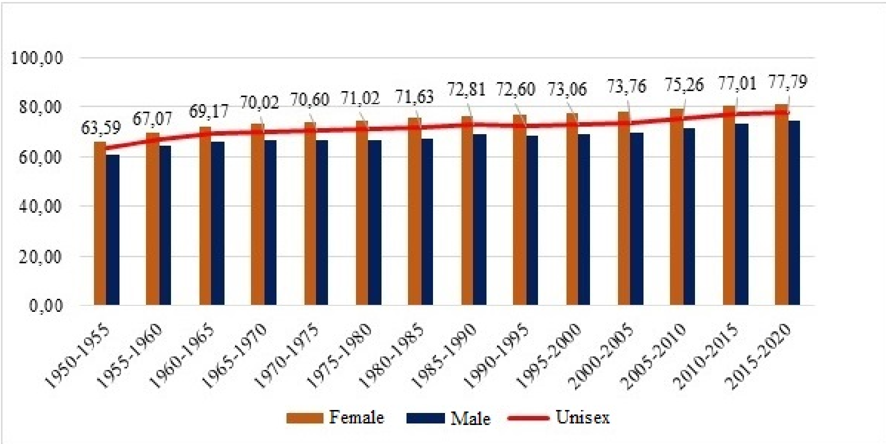

FIGURE 2.1: European Life Expectancy at Birth for Male, Female and Overall

Many academics, Yang et al. (2010), Deng et al. (2012) and D'Amato et al. (2014), among others, discuss improvements in life expectancy. This reflects that people are living much healthier and better now as compared to previous generations. According to Gaille and Sherris (2010); Roy (2012); and French and O'Hare (2014), some potential reasons for these improvements may be socio-economic status, marital status, technological advancement in medicine, nutrition and diet, lifestyle and living environment. The increases in life expectancy are mainly due to improvement in the food supply, sanitation, technology, basic healthcare and education (Roy, 2012). It is believed that the evolution of the trend in longevity has produced a new curve to the standard life expectancy, making current life tables underestimate future life expectancy (Barrieu et al., 2012). This phenomenon has become more crucial since it has drawn the attention of many organisations like pension funds, life and health insurers and individuals.

Holders of longevity risk, particularly now need to be more mindful of their funding and price methods to ensure their products are priced and reserved for sufficiently (Richards and Currie, 2009; Seklecka et al., 2017a). The government faces challenges on financing an ageing population with longer life spans whilst the insurers

[1]Source: United Nations Department of Economic and Social Affairs / Population Division World Population Prospects: The 2015 Revision, Volume II: Demographic Profiles.

liable for providing conducive and appropriate healthcare solutions need to understand expected lifetimes sufficiently. The threats of longevity risk also extend to individuals where they have not, or are unable to purchase products that insure their retirement income. Eventually, the longevity phenomenon will affect the annuity values, pensions, insurances and individual savings (Richards and Currie, 2009; Godínez-Olivares et al., 2016; Seklecka et al., 2017a). Thus, the need to model mortality more accurately becomes critical in planning for one's sustainability. It helps the government in funding pension policies sufficiently, actuaries and insurers in pricing age-related financial instruments appropriately and individuals in planning their post-retirement favourably.

The challenge of modifying mortality projections is made more interesting since the mortality improvements are not comparable across all countries. Roy (2012), in his study, found that different countries experience different stages of demographic transition due to different patterns of population growth. He further commented that this scenario can be clearly observed when comparing the developed and emerging countries. Thus, an accurate identification of factors affecting the mortality experience in a particular country is crucial. On the other hand, evidence of a relationship between economic changes and mortality in many developed and developing countries has been presented by many researchers (Preston, 2007; Granados, 2008; Hanewald, 2011; Granados and Ionides, 2011; Hanewald et al., 2013; Niu and Melenberg, 2014; Rolden et al., 2014; Boonen and Li, 2017).

This study analyses the effect of socio-economic factors on mortality trends for selected Eurozone countries namely, Austria, Belgium, Germany, Greece, France, Italy, the Netherlands, Portugal and Spain. These countries are considered in this study not only because they are members of the same monetary union, but also due to their populations exceeds 10 million citizens.[2] According to Chen et al. (2017), small population is bias in modelling mortality stochastically. Empirical research has found that mortality rates of smaller populations exhibit significantly more variability compared to the observed rates in larger populations. Another related issue is that empirical data from smaller populations might only be available for a relatively short period, which makes mortality projections rather uncertain. As a result, this study focuses on the Eurozone countries that have a significant number of the population size.

[2] 2011 census.

Additionally, the majority of these countries has shared common monetary and fiscal policies for the last 40 years (even before being members in the Eurozone system), and also due to their co-dependent and homogeneous growth cycles (Chen and Mills, 2009). Despite having an overall competitive economic growth performance, different countries in Eurozone experience different pace of economic growth. Countries that have significant economic growth are mainly those located in central Europe like Austria, Belgium, Germany, France and the Netherlands. Whilst gradual growth is experienced by those representing the peripheral ones like Spain, Greece, Italy and Portugal.

One of the main goals of this study is to examine the relationship between trends in mortality and trends in GDP change (as a proxy for economic fluctuations). In this direction, a study conducted by Niu and Melenberg (2014) gave impetus to us to extend it on the selected Eurozone countries. First, we explore the relationship between the time-dependent factor of the Lee and Carter (1992) model k_t^1, and the logarithm of GDP for males and females aged between 0 to 89 (similar for unisex data). Then, this study investigates the long run relationship and correlation between these factors using Johansen (1988) co-integration test. Additionally, this study examines the correlation between both factors using appropriate correlation tests. For instance, Pearson (1895)'s correlation coefficient test shows that there is a long-run relationship between the mortality index and GDP as we observe a strong negative correlation between them. The findings from these tests provided a strong foundation and motivation to investigate this relationship further. Finally, we introduce a new stochastic central mortality rates model by modifying O'Hare and Li (2012) model (hereafter referred to as the OL) through the implementation of an additional GDP-related factor capturing the effect of economic fluctuations on mortality.

Forecasting mortality rates is a natural application of our approach. As in previous studies (Niu and Melenberg, 2014; Seklecka et al., 2017a,b), among others, our forecasting methodology is a combination of the explanation and extrapolation methods (Booth and Tickle, 2008) (see Section 2.2 for more details). Analytically, the real GDP per capita is included as an observable factor, which captures the correlation between long-term trends in mortality dynamics and economic growth. The trend in mortality rates is captured and the future mortality rates are based on historical trends.

The remainder of this chapter is organised as follows. Expectation, explanation and extrapolation of life expectancy are discussed in Section 2.2. Whereas, Section 2.3 briefly reviews the main mortality models and this study further investigates the relationship between economic growth (GDP) and mortality rates. Section 2.4 focuses on the statistical analyses of the data for the nine Eurozone countries (in our study, Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain), derived from the Human Mortality Database (HMD), followed by a descriptive analyses of the long run relationship among time series data using the Johansen (1988) co-integration test, and other various correlation tests. Based on the findings and comparison studies, this study proposes a new GDP-related model in Section 2.5, with detailed discussion of the fitting process. The forecasting performance using ARIMA process and Exponential Smoothing is discussed in Section 2.6. Finally, Section 2.6 concludes the whole discussion of the approach.

2.2 Expectation, Explanation and Extrapolation of Life Expectancy

Rapid increases in life expectancy have increased the desire and need to study mortality. As changes in mortality experience is not comparable across all countries, a dynamic approach that can accurately describe the changes is essential. According to Booth and Tickle (2008), mortality forecasting can be divided into three categories; expectation, explanation and extrapolation. The expectation approach is based on the subjective opinions of experts, the explanation approach is derived from certain causes of death with known determinants whilst the extrapolation approach makes use of the regularity found in age patterns and trends over time. It estimates the future mortality using the current mortality rates and estimates the rate of change of future mortality by observing patterns in past changes. Most of the models in the literature are based on the extrapolation method.

2.2.1 Extrapolative Mortality Modelling

The evolution of mortality studies has started a few centuries ago. A number of pioneering works were carried out as early in the 15th century. In 1693, a famous astronomer, Edmond Halley developed the first life table based on applications to life contingencies. Subsequently, in 1725, Abraham De Moivre proposed the first mathematical formula of mortality modelling.

$$l_x = k\left(1 - \frac{x}{86}\right), \; for \; 12 \leq x \leq 86 \tag{2.1}$$

where l_x is the number of individuals still alive at age x last birthday from an original pool, l_0 of individuals, and k is a constant. He assumed that all individuals must die before the age of 86. For more details, please see Bowers et al. (1997).

A century later, Gompertz (1825) proposed the new mortality law in 1825. Theoretically, the Gompertz's idea on the term of $force$ of mortality can be accurately expressed in the model. Denoting the force of mortality by μ_x, Gompertz's law is as follows:

$$\mu_x = \alpha \exp\left(\beta x\right) \tag{2.2}$$

where α and β are positive parameters and x denotes the age[3]. More discussions about mortality laws can be found in Bowers et al. (1997). What is more, Heligman and Pollard (1980) proposed a class of formulae which aimed to represent the age pattern of mortality over the whole span of life[4].

Over 160 years later and with advances in computing techniques, Lee and Carter (1992) proposed the new mortality model that has been widely accepted and referred to in mortality studies. They remarkably proposed a simple stochastic model reflecting the reduction of the annual log age-specific death rates through a time-dependent index,

$$\ln(m_{x,t}) = b_x^1 + b_x^2 k_t^1 + \epsilon_{x,t} \tag{2.3}$$

where $m_{x,t}$ is the central death rate, the ratio between the number of people aged x who died in year t, and the exposure to risk of the average population aged x in year t. Factor b_x^1 describes the average age-specific mortality, that ensures the basic shape of the mortality curve over ages is in line with historical observations,

[3]By focusing on the old ages, Gompertz (1825) has weakened the model by not representing the mortality over the whole lifetime span.

[4]Heligman and Pollard (1980) proposed a sum of three terms representing different components of mortality:

$$m(x) = A^{(x+B)^C} + De^{E(\ln x - \ln F)^2} + \frac{GH^x}{(1+GH^x)},$$

where $A, B, ..., H$ are parameters to be estimated.

factor k_t^1 represents the changes in the mortality level, whilst factor b_x^2 describes the decline in mortality at age x. It explains how rapidly/slowly mortality rates decline in response to k_t^1. The error term at age x and time t, is $\epsilon_{x,t}$. The LC model describes the smooth and gradual decline of mortality rates over time. This has become debatable as the variance of mortality rates exponentially grows in time. Following the LC model, a series of extensive studies were conducted by many researchers including Lee and Miller (2001), Booth et al. (2002), Renshaw and Haberman (2006), Hyndman and Ullah (2007), Cairns et al. (2009, 2011), Plat (2009), O'Hare and Li (2012) building on the extrapolation approach and improving the mortality modelling.

To be more precise, Renshaw and Haberman (2006) modified the LC model by adding a cohort effect parameter into the formula:

$$\ln(m_{x,t}) = b_x^1 + b_x^2 k_t^1 + b_x^3 \gamma_{t-x} + \epsilon_{x,t} \tag{2.4}$$

where γ_{t-x} models the cohort effect. Their model provides a better fit to the historical data, where the cohort effect was observed in the past in a particular country with the best results for $b_x^3 = 1$. However, this model suffers from lack of robustness and has a trivial correlation structure (Cairns et al., 2009, 2011). Notwithstanding this criticism, Haberman and Renshaw (2011) then conquer this argument by justifying that issues observed in these studies were a result of the fitting procedure used to obtain parameter estimates. This statement has also been confirmed in a more recent paper by Hunt and Villegas (2015).

Particularly, Cairns et al. (2006b) proposed a two-factor model of mortality:[5] Subsequently, in their extended research using data from England and Wales, and United States, they observed that the fitted cohort effect appears to have a trend in the year of birth. This suggested that the cohort effect compensates for a lack of a second age-period effect as it attempts to capture the cohort effect from the data. Hence, in 2009, Cairns et al. improved the the two-factor model by adding

[5] Cairns et al. (2006b):

$$\log\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = k_t^1 + (x-\bar{x})k_t^2 + \epsilon_{x,t},$$

where q_x is the probability that a person aged x dies within the next year ($q_{x,t} \approx 1 - e^{-m_{x,t}}$), $\bar{x}$ is the mean age in the sample range and (k_t^1, k_t^2) are assumed to be a bivariate random walk with drift.

the second age cohort effect. This improvement captures the cohort effect as an additional effect on top of the two-factor (age and period) effects.

Following this, Plat (2009) incorporated the cohort and age-period effects[6] to the LC model. His model provides significant and better results since it implies the importance of younger ages in modelling the mortality experience of a population.

Subsequently, O'Hare and Li (2012) extended the Plat (2009) model by adapting the non-linear profile of mortality at lower ages:

$$\ln(m_{x,t}) = b_x^1 + k_t^1 + (\bar{x} - x)k_t^2 + \big((\bar{x} - x)^+ + ([\bar{x} - x]^+)^2\big)k_t^3 + \gamma_{t-x} + \epsilon_{x,t} \quad (2.5)$$

where k_t^2 factor allows changes in mortality to vary between ages reflecting the historical observation that improvement rates can differ for different age classes, k_t^3 models the effects specific to the lower ages, γ_{t-x} models the cohort effect, $(\bar{x} - x)^+ = max(\bar{x} - x, 0)$, and $\bar{x}$ is the average of age considered. Their model provides a better fit for the range of countries considered and shows flexibility to fit the mortality rates of a wider range of ages. What is more important, this model does not lose any of the benefits of the previous stochastic models.

The number of wide-ranging mortality studies has increased significantly due to the rapid increase of longevity and the need to better understand it. That increase in all age-intervals is primarily due to several factors such as socio-economic, medical improvements, lifestyle, living environment, climate change and many others (Granados, 2008; Granados and Ionides, 2011; Hanewald, 2011; French and O'Hare, 2013, 2014; Niu and Melenberg, 2014; Cairns et al., 2016; Seklecka et al., 2017b). Socio-economic factor has been discussed and agreed widely as one of the important factors that affect longevity (Bhargava et al., 2001; Batchvarov and Gakwaya, 2006; Renton et al., 2012; Preston, 2007; Granados, 2008; Granados and Ionides, 2011; Hanewald, 2011). According to the Institute and Faculty of Actuaries (IFoA), as socio-economic status increases, the life expectancy is also increases. Among other things, socio-economic status can affect a person's ability to access adequate medical care and participate in healthier lifestyle habits like exercising more, smoking less and maintaining a healthy weight.

[6] Plat (2009) model specification is given by

$$\ln(m_{x,t}) = b_x^1 + k_t^1 + (\bar{x} - x)k_t^2 + (\bar{x} - x)^+ k_t^3 + \gamma_{t-x} + \epsilon_{x,t},$$

where $(\bar{x} - x)^+ = \max(\bar{x} - x, 0)$, and $\bar{x}$ is the average of the ages considered.

In accessing the growth of economics and identifying long-term trends in a particular country, factors like GDP, unemployment rate, income and wages, consumer price index, currency strength and interest rates are among those indicators that being studied in observing how the economy changes over time. These indicators are also known as lagging indicators, reflect the economy's historical performance and changes. These indicators are only identifiable after an economic trend or pattern has already been established. Among these indicators, changes in GDP has been widely used by many economists in measuring the economy's current health in a particular country. When GDP increases, it is a sign the economy is strong. Moreover, GDP is a key determinant to most businesses as to decide and adjust their expenditures on inventory, payroll, and other investments based on GDP output.

This study analyses GDP as one of those influencing factors that affect mortality. This study employs a correlational studies between GDP and mortality. Correlational studies is more important and useful than other methods as it can be widely used for many variables especially for variables that cannot be simply manipulated for ethical reasons like human malnutrition. Additionally, as this study involves with variables that definitely cannot be manipulated like birth, sex and age, thus the application of correlational studies is deemed appropriate as the scientific knowledge concerning them must be based on correlation evidence.

2.2.2 Economic Growth and Mortality Rates

Economic growth and health status are two major factors being deeply studied in gauging nations' social development and public policy. Socio-economic factors such as GDP, inflation and unemployment have clearly been observed to have a causal effect on mortality experience. The majority of the related studies showed that the improvements in mortality have been accompanied by growth in GDP. Bhargava et al. (2001) claimed that there was a positive effect between adult survival rates and GDP growth rates for low-income countries. Renton et al. (2012) claimed that GDP makes a contribution to changes in health indicators through GDP growth and the increased elasticity of health indicators with GDP over time. While Preston (2007) suggests that in the long run, higher economic output results in lower mortality.

On the other hand, Granados (2008); Granados and Ionides (2011); Granados (2012) conducted various studies related to macroeconomic fluctuations and mortality. Granados (2008); Granados and Ionides (2011) studied correlations of GDP, unemployment, and the labour force participation ratio with crude, age-and-sex specific and cause-and-sex specific mortality rates in postwar Japan and contemporary Sweden. Subsequently, Granados (2012) studied economic growth and health progress in England and Wales, and found that there was a negative relationship between economic growth and life expectancy at birth.

In the meantime, Hanewald (2011) investigated the effects of macroeconomic fluctuations and trends in the causes of death on the period effect k_t^1 in the LC model based on the data for six OECD countries (Australia, Canada, Japan, the Netherlands, the United Kingdom, and the United States) over the period of 1950–2006. Analysis presented in the paper shows that the time-dependent indicator correlates significantly with macroeconomic fluctuations. The results also indicate a reduction of the mortality index when the economy strengthens. Further analysis found that these correlations were more significant from a long-run period effect.

Following this, Niu and Melenberg (2014) adopted the idea to study the relationship between latent factors in mortality and macroeconomic variables, based on the data for six industrialised countries, the United States, the United Kingdom, the Netherlands, Canada, Australia, and Japan over the period of 1950–2007. As a result of their investigation, a new mortality model was proposed. Niu and Melenberg (2014) incorporated the economic factor (real GDP per capita as proxy) in their extended model from Lee and Carter (1992):

$$\ln(m_{x,t}) = b_x^1 + b_x^2 k_t^1 + d_x g_t + \epsilon_{x,t} \tag{2.6}$$

where the newly included parameter d_x denotes the real GDP per capita in logarithm. They emphasised that the proposed model can generate more interpretable scenarios about future longevity based on the forecast of future economic growth.

What is more, French and O'Hare (2014) found the correlations between the latent factor structure within mortality data and a selection of health and economic factors in their study. The inclusion of exogenous determinants of health factors such as gross domestic product (GDP), health expenditure and lifestyle-related risk provide better forecasts in the countries under study, the United States, the United Kingdom, Japan, Finland and the Netherlands.

More recently, Cairns et al. (2016) introduced a flexible multi-population, gravity-type mortality model for older Danish Males. The population was divided into ten sub-groups[7] using a new affluence index that combines wealth and income reported on the Statistics Denmark national register database. The model provides reasonable forecasts of mortality rates that preserve the sub-group rankings at all ages.

In the very recent study conducted by Boonen and Li (2017), the inclusion of the economic growth (represented by GDP per capita) signified to provide a better in-sample fit and out-of-sample forecast performance for each group of countries observed. Their multi-population model[8], generates lower (higher) forecasted life expectancy period for countries with high (low) GDP as compared to another mortality model under study.

Concurrently, various studies were conducted to investigate the effect of macroeconomic fluctuations on mortality in different socio-economic groups for various EU countries such as the United Kingdom, Germany, the Netherlands, Spain, Greece, Finland, Iceland, Denmark or Slovakia (Rolden et al., 2014; Granados and Rodriguez, 2015; Regidor et al., 2016; Cairns et al., 2016; Rosicova et al., 2016). However, only a few authors proposed a new mortality model that incorporates the identified exogenous economic factors; see for example (French and O'Hare, 2014; Niu and Melenberg, 2014; Boonen and Li, 2017).

In the next section, this study follows the idea presented by Niu and Melenberg (2014) and examines the relationship between GDP and mortality rates for those nine most populous Eurozone countries.

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[7] For i subgroup they proposed the following formula,

$$\log(m_{i,t,x}) = \beta_0^{(i)}(x) + k_1^{(i)}(t) + (x - \bar{x})k_2^{(i)}(t).$$

[8] The general model considered by Boonen and Li (2017) is given by

$$\log m_{i,x,t} = a_{i,x} + \sum_{j=1}^{J} B_{j,x}K_{j,t} + \sum_{l=1}^{L} \gamma_{l,x}g_{l,t} + b_{i,x}k_{i,t} + \epsilon_{i,x,t},$$

$$\epsilon_{i,x,t} \overset{i.i.d}{\sim} N(0, \sigma^2_{\epsilon_{i,x}}),$$

where $a_{i,x}$,$b_{i,x}$, $k_{i,t}$ are the population-specific parameters,$K_{j,t}$ is the j-th common latent mortality trend, $g_{l,t}$ is the l-th principal component of GDP for the I populations, and $B_{j,x}$ and $\gamma_{l,x}$ are their respective age-specific loadings.

2.3 Mortality and GDP

This section describes the dataset used in this study and thereafter, observing the trends in mortality index in respect to the macroeconomic factor (GDP) fluctuations. Therefore, this study establishes that there is significant correlation between mortality rate and economic growth.

2.3.1 Data from Selected Eurozone Countries

The mortality data is obtained from the Human Mortality Database (HMD)[9] for years 1960–2012[10]. In line with the discussed literature, we estimate the mortality experience only for the nine most populous Eurozone countries namely Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The estimations are based on the three selected mortality models, namely Lee and Carter (1992) (LC), O'Hare and Li (2012) (OL), and Niu and Melenberg (2014) (LC-GDP). Data for male, female and unisex[11] have been analysed separately as they portray different patterns of mortality.

As for the GDP data, our analysis is based on the yearly GDP per capita (constant LCU)[12] performance for these countries over the period of 1960–2012[13]. All data were downloaded from the World Bank website[14]. The countries were then categorised into two groups based on the GDP trends that they experienced. As depicted in Figure 2.2, similar trends were observed among the members of the two groups of countries; **Group A**: Austria, Belgium, Germany, France and the Netherlands; and **Group B**: Spain, Greece, Italy and Portugal.

The GDP performance for Austria, Belgium, Germany, France and the Netherlands (see Figure 2.2) show a promising growth over the considerable period. The

[9]The Human Mortality Database, which was furnished by the University of California, Berkeley (USA), and Max Planck Institute for Demographic Research (Germany), provides a detailed mortality and population data accessible online at www.mortality.org or www.humanmortality.de (2017).

[10]Except Greece, where data is available from 1981

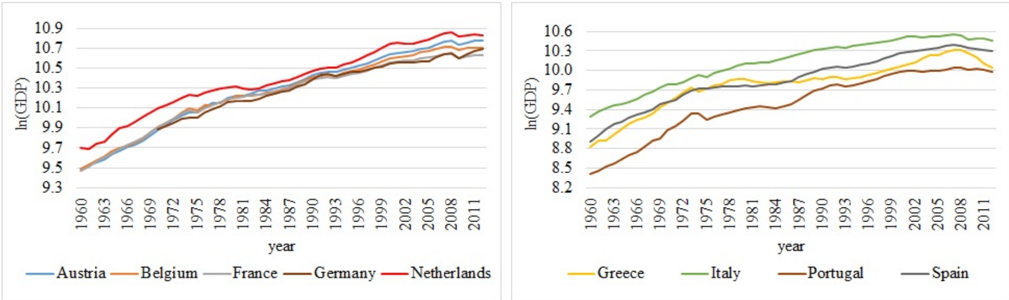
[11]In other words, it is suitable for both sexes. In this study, the calculations are based on the total number of deaths and exposure to risk.

[12]GDP is the sum of gross value added by all resident producers in the economy plus any product taxes and minus any subsidies not included in the value of the products. It is calculated without making deductions for depreciation of fabricated assets or for depletion and degradation of natural resources. All data are in constant local currency.

[13]Except Germany, where data is available from 1970.

[14]`http://data.worldbank.org/`

momentum of GDP growth keeps on improving, and significant growth can be seen clearly upon the Euro (€) adoption in 1999.



(A) Log of GDP for Austria, Belgium, France, Germany and Netherlands.

(B) Log of GDP for Greece, Italy, Portugal and Spain.

Figure 2.2: Logarithm of GDP for Austria, Belgium, France, Germany, Greece, Italy, Netherlands, Portugal and Spain. Data from 1960 till 2012 (except Germany, 1970–2012).

Meanwhile, the GDP performance for Spain, Greece, Italy and Portugal (see Figure 2.2) also show an increasing trend over a considerable period. Despite having a gradual growth over the period, the performance of these four countries is remained to be the same upon the adoption of Euro (€) in 1999. A study conducted by Çiftçioğlu and Betyak (2014) on the crisis countries revealed that the monetary union of the Eurozone did not automatically ensure a higher rate of economic growth among its members. They further commented that the dynamics of economic growth were ultimately driven by macroeconomics. This remark seems to agree with the earlier findings of Drake and Mills (2010) and Giannone et al. (2011) that the adoption of the Euro (€) did not bring significant jump in economic growth to all Eurozone countries. However, besides keeping the same pace of economic growth, these crisis countries have been outperformed remarkably since joining the Eurozone. Following the union, they have been bailed out successfully through the implementations of structural reforms like improving public finances, reducing deficits, and cutting labour costs.

2.3.2 Trends in Mortality Index and GDP Fluctuations

This study applies a similar technique to that of Niu and Melenberg (2014) in examining the relationships between the LC mortality index k_t^1 and the macroeconomic fluctuations. Like previous studies, we also agree that there is an inverse relationship between the mortality index k_t^1 and the positive economic growth. In our study, based on the LC model parameters, we analyse the trend of the mortality index k_t^1 and the logarithm of GDP, separating the data for male, female and unisex for each Eurozone country, accordingly.

This study first applies the Perron (1988) test for each dataset (males, females, unisex) of the mortality and the logarithm of GDP performances for each country. Results for male, female and unisex are similar (see Table 2.1). In all cases, there is no evidence to reject the null hypothesis at the 1% level of significance level except for the unisex result of Belgium.

Table 2.1: Results of Phillips - Perron (PP) Test

	male	**female**	**unisex**	**ln(GDP)**
Austria	3.996	2.888	3.039	-3.541**
Belgium	3.469	0.700	-7.428	-3.676***
France	3.459	0.916	1.986	-5.419***
Germany	2.523	1.528	1.844	-1.834
Greece	1.039	0.542	0.751	3.237
Italy	6.087	1.272	3.073	-6.029***
Netherlands	6.655	-0.256	2.593	-1.609
Portugal	2.956	2.072	2.479	-2.962**
Spain	1.728	0.697	0.962	-3.425**

Notes: $^{*}p < 0.10, ^{**}p < 0.05, ^{***}p < 0.01$.

However, for the logarithm of GDP, this study may reject the null of non-stationarity[15] for Austria, Portugal and Spain at the 5% significance level and Belgium, France

[15] Non-stationary data, as a rule, are unpredictable and cannot be modelled or forecasted. The results obtained by using non-stationary time series may be spurious in that they may indicate a relationship between two variables where one does not exist. In order to receive consistent, reliable results, the non-stationary data needs to be transformed into stationary data.

and Italy at the 1% significance level, respectively.To support this analysis, this study then applies the Kwiatkowski et al. (1992) test (KPSS) on the same dataset. As stated in Table 2.2, there is evidence to reject the null hypothesis that an observable time series is stationary around a deterministic trend for all cases as all results are greater than the significance level of 1%. Therefore, this study proceeds under the assumption of the presence of a unit root in the time series. Subsequently, this study performs the Johansen (1988) cointegration test. This is to analyse the cointegration between the mortality index and the GDP performance.

TABLE 2.2: Results of Kwiatkowski - Phillips - Schmidt - Shin (KPSS) Test

	male	**female**	**unisex**	**ln(GDP)**
Austria	1.253	1.280	1.274	1.269
Belgium	1.264	1.298	1.289	1.264
France	1.282	1.302	1.295	1.258
Germany	0.962	0.997	0.990	1.048
Greece	0.971	0.982	0.977	0.897
Italy	1.255	1.296	1.283	1.260
Netherlands	1.199	1.266	1.281	1.256
Portugal	1.288	1.297	1.293	1.236
Spain	1.296	1.302	1.301	1.252

Table 2.3 presents the results of the Johansen's test. The null of no cointegration ($r = 0$) is only rejected for male, Germany at the 5% significance level, but the null of one cointegration vector ($r \leqslant 1$) is mostly rejected for male, female and unisex for the majority of the countries at the 10% significance level. In other words, Johansen cointegration tests indicate that the two series have a long-run relationship. The analysis conducted above and as presented in Tables 2.1, 2.2 and 2.3, indicates a possible long run relationship between the macroeconomic indicator (GDP) and the mortality rates in the countries under consideration, and thus further analysis is conducted in the following sections.

TABLE 2.3: Results of Johansen Cointegration Test

	male		**female**		**unisex**	
	$r \leqslant 1$	$r = 0$	$r \leqslant 1$	$r = 0$	$r \leqslant 1$	$r = 0$
Austria	19.39**	42.52	19.65**	38.85	19.45**	41.31
Belgium	26.99	34.97	20.35	29.91	24.59	30.83
France	22.37	33.80	20.17***	25.06	21.98	27.51
Germany	5.64*	12.69**	7.41*	19.11	7.47*	21.52
Greece	4.49*	18.21	4.87*	19.08	4.70*	18.88
Italy	17.23***	49.69	13.32*	43.69	15.30**	46.65
Netherlands	22.76	33.75	7.68*	23.61	16.08***	24.27
Portugal	9.96*	34.06	9.08*	40.34	0.47*	36.99
Spain	7.69*	40.00	7.83*	50.26	7.79*	45.05

Notes: $^{*}p < 0.10,^{**} p < 0.05,^{***} p < 0.01$. Critical values for the null of $r = 0$ are: 7.52 for $p < 0.10, 9.24$ for $p < 0.05$, and 12.97 for $p < 0.01$. Critical values for null of $r \leqslant 1$ are: 13.75 for $p < 0.10$, 15.67 for $p < 0.05$, and 20.2 for $p < 0.01$.

2.3.3 Correlations between Mortality Index and GDP Performance

This study conducts correlation studies between changes of LC mortality index k_t^1 separated by male, female and unisex with GDP fluctuations through the applications of Pearson (1895)[16], Kendall (1938)[17] and Spearman (1904)[18] tests. As presented in Table 2.4, this study observes highly significant negative correlations for all nine countries with respect to all panels, i.e., male, female and unisex. Additionally, Spearman test provides the highest correlations for all panels of male, female and unisex across all countries.

[16]Pearson's correlation coefficient is the covariance of the two variables divided by the product of their standard deviations. The form of the definition involves a "product moment", that is, the mean (the first moment about the origin) of the product of the mean-adjusted random variables; hence the modifier product-moment in the name.

[17]It is a measure of rank correlation: the similarity of the orderings of the data when ranked by each of the quantities.

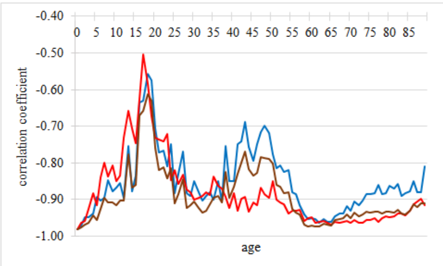
[18]Spearman's rank correlation coefficient is a nonparametric measure of statistical dependence between two variables. It assesses how well the relationship between two variables can be described using a monotonic function.

Table 2.4: Correlation Coefficients of Pearson, Kendall and Spearman (male, female and unisex data).

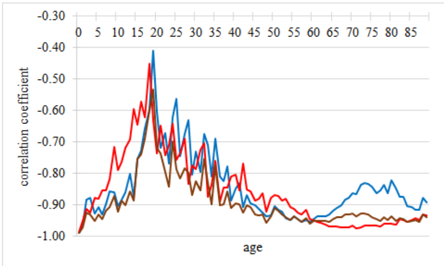
	male			**female**			**unisex**		
	Pearson	Kendall	Spearman	Pearson	Kendall	Spearman	Pearson	Kendall	Spearman
Austria	-0.9205234	-0.9485816	-0.9896874	-0.9455686	-0.9609929	-0.9950065	-0.9386697	-0.9574468	-0.9930525
Belgium	-0.9198888	-0.9414894	-0.9865393	-0.9592453	-0.962766	-0.9955493	-0.945081	-0.9574468	-0.9925098
France	-0.9365780	-0.9680851	-0.9954407	-0.9639948	-0.9680851	-0.9973947	-0.9522943	-0.9680851	-0.9968519
Germany	-0.9017752	-0.8719772	-0.9632345	-0.9259671	-0.9459459	-0.9921217	-0.917574	-0.940256	-0.9912463
Greece	-0.9669239	-0.9031339	-0.9774115	-0.9558699	-0.9088319	-0.979243	-0.9612304	-0.9031339	-0.9774115
Italy	-0.9071805	-0.9716312	-0.9973947	-0.9570624	-0.9769504	-0.9982631	-0.9372254	-0.9751773	-0.9978289
Netherlands	-0.8900065	-0.8687943	-0.9640686	-0.9590814	-0.9255319	-0.9895788	-0.9472684	-0.9361702	-0.9913157
Portugal	-0.9343509	-0.9450355	-0.9931611	-0.9440716	-0.9539007	-0.9938124	-0.939829	-0.9503546	-0.9934868
Spain	-0.9604133	-0.9663121	-0.9968519	-0.9632196	-0.964539	-0.9966348	-0.9634361	-0.962766	-0.9965263

As for the correlations between average mortality rates at individual ages with GDP fluctuations, the majority of the countries except Greece are giving strong correlations for all male, female and unisex. The peculiar results for Greece might be due to the smaller range of data as compared to other Eurozone countries. For male data (see Figure 2.3, blue curves), countries such as Portugal, Spain, Belgium and France show weak correlation for ages ranging from 15–35 years old.

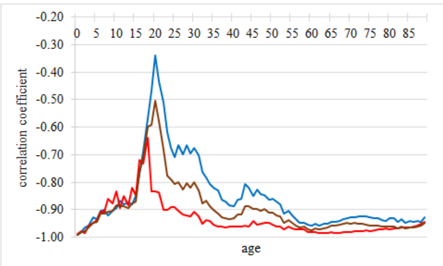
Strongest correlations are observed for almost all countries on 0-15 years old and above 40 years old populations. For female data (see Figure 2.3, red curves), countries such as Austria, Belgium, the Netherlands and France are less correlated within the age range of 10–55 years old. The strongest correlations are observed for ages 0–5 years old and above 55 years old. As for unisex (see Figure 2.3, brown curves), producing quite similar results with the male data, they are highly correlated from 0–15 years old and above 40 years old with the score of −0.95.



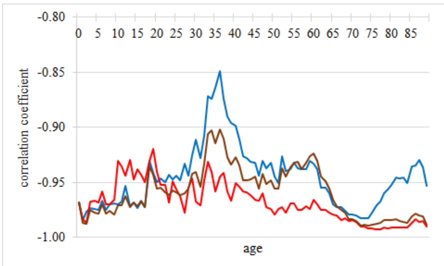
(A) Austria



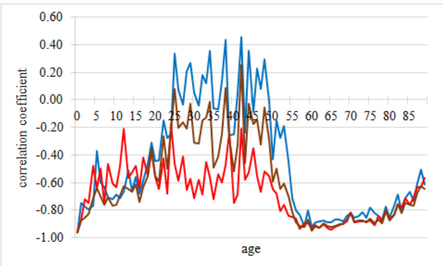
(B) Belgium



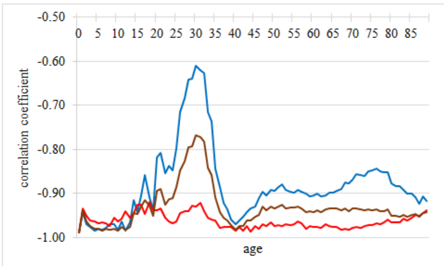
(C) France



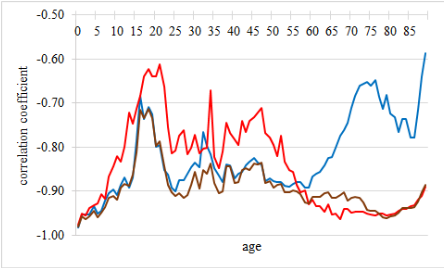
(D) Germany



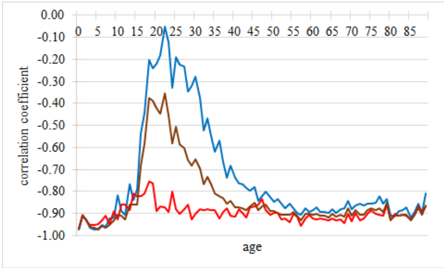
(E) Greece



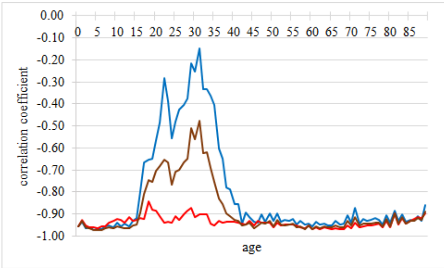
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

FIGURE 2.3: Correlation coefficients between the logarithm of GDP and mortality rates for male (blue), female (red) and unisex (brown) for nine countries: Austria, Belgium, France, Germany, Greece, Italy, Netherlands, Portugal and Spain. Data from 1960 to 2007 (except: Germany: 1970–2007, Greece: 1981–2007).

As findings compare well with previous studies, this study finds that correlations for older ages are extremely strong regardless of gender. Moreover, the correlations fluctuations for ages above 50 is quite stable and firm. These results provide some confidence that incorporating these important findings in the new model of mortality is a worthwhile exercise.

This study includes the idea of incorporating the economic growth fluctuations into the mortality model. However, differently from Niu and Melenberg (2014), this study does not incorporate the observable factor of GDP directly into the model. Based on the analysis of individual ages, this study includes the GDP factor in relation to the age dependent, which reflects the impact of GDP on the individual age. What is more, this study keeps all the good aspects of the LC model extensions (like cohort effect introduced by Renshaw and Haberman (2006), age-period by Cairns et al. (2006a), second period by Plat (2009), and quadratic effect by O'Hare and Li (2012). For that reason, this study incorporates the age dependent GDP related factor into the OL model. By doing so, this study aims to provide another alternative perspective on understanding the various models of mortality modelling. Based upon the findings of the analyses, this study formulates a new model incorporating the GDP factor. In the next section, this study proposes the new model.

2.4 OL-GDP Model for Central Mortality Rates

Recently, a large number of new mortality models have been proposed to analyse the performance of fitting the historical mortality rates and ability to project more plausible mortality rates for the future. Many of these suffer from being over-parametrised or have terms added in an ad-hoc manner which could not be justified in terms of demographic significance. In addition, poor specification of a model may attribute to incorrect cohort effects that will eventually lead to implausible projections of the mortality rates. However, this study discovers that the presence of a macroeconomic factor like GDP can implicitly represent a demographic change on the demography (see Section 2.3 for details). This study extends the procedures of O'Hare and Li model by constructing a new mortality model using a combination of a toolkit function, *Lifemetrics* in R code and experts judgement. By following the procedure, this study evaluates the performance of the new model against the alternative models using robustness and statistical significance tests. This study demonstrates the data from the nine most populous countries in the Eurozone. Apparently, this procedure produces a relatively parsimonious model that ideally fits the data well.

2.4.1 The proposed GDP-related model

To further assess the role of economic growth on mortality dynamics, this study presents the new model fitting the GDP-related factor on the mortality index using the OL model. Thus, that additional factor, $\mathbf{cg_x}$ is a vector of correlation coefficients between the logarithm of GDP per capita and mortality rates for ages 0–89.

$$\ln(m_{x,t}) = b_x^1 + k_t^1 + \mathbf{cg_x}(\bar{x} - x)k_t^2 + (\bar{x} - x)^+ k_t^3 + ([\bar{x} - x]^+)^2 k_t^4 + \gamma_{t-x} + \epsilon_{x,t} \quad (2.7)$$

The original model of O'Hare and Li (2012), is an extended version of the LC model. By keeping all the good aspects of the LC model, the OL model provides new good features for better fitting and performance. This study observes similar factors of b_x^1, k_t^1 as the original OL model while factor k_t^2 allows varieties of ages reflecting the historical observation of rates improvement for different ages, including also the impact of economic fluctuations. Factors k_t^3 and k_t^4 capture the effect of young age and the presence of non-linearity respectively.

This study conducts a comparative analysis between the proposed new model against three other mortality models namely, LC, OL and the extended LC model with GDP (hereafter referred to as the LC-GDP model) by Niu and Melenberg (2014). This study indicates the new model as the extended version of the OL model with GDP effect (hereafter referred to as the OL-GDP model).

2.4.2 Estimation and Quality of Fitting

Models with multi-parameters are commonly exposed to the identifiability problem. Like other multi-parametrised models, the OL-GDP is also experiencing the same problem. Each parameter of the model tends to identify similar values of the function of $\ln(m_{x,t})$. For more details see Plat (2009).

$$\begin{cases} \tilde{\gamma}_{t-x} = \gamma_{t-x} + \psi_1 + (t-x)\psi_2 \\ \tilde{k}_t^1 = k_t^1 - \psi_1 - (d\bar{x} + t)\psi_2 \\ \tilde{k}_t^2 = k_t^2 + d\psi_2 \\ \tilde{b}_x^1 = b_x^1 + (1+d)x\psi_2 \end{cases} \tag{2.8}$$

where ψ_1, ψ_2 and d are constants.

Apparently, the identifiability problem can be resolved by setting the identifiability constraints. Since the proposed model is a derivation of Plat (2009), O'Hare and Li (2012), this study adopts their approach in setting up the constraints. At this juncture, this study undertakes a similar approach to Cairns et al. (2009), by applying some additional constraints to the model[19]. Thus,

[19] Cairns et al. (2006a) model has simpler structure, thus requiring fewer constraints.

$$\begin{cases} \sum_{c=c_0}^{c_1} \gamma_c = 0 \\ \sum_{c=c_0}^{c_1} c\gamma_c = 0 \\ \sum_t k_t^3 = 0 \\ \sum_t k_t^4 = 0 \end{cases} \tag{2.9}$$

where c_0 and c_1 are the earliest and latest year of birth to which a cohort effect is fitted, where $c = t - x$. The last two constraints are only used to normalise the estimates for factors: k_t^3 and k_t^4.

In fitting the mortality rates, Lee and Carter (1992) used Singular Value Decomposition (SVD) methodology. Then, Brouhns et al. (2002) improved the fitting of the LC model by describing the number of deaths $D_{x,t}$ as a Poisson distribution with parameter $(E_{x,t}m_{x,t})$. It is notable that the Brouhns et al. (2002) approach is more plausible than SVD as it caters for heteroscedasticity of the mortality rates for different ages. Indeed, their method has been adopted by many other authors (Renshaw and Haberman, 2006; Cairns et al., 2009; Plat, 2009; O'Hare and Li, 2012; Seklecka et al., 2017a). For this reason, this study follows the approach from the corresponding literatures. By adopting the Brouhns et al. (2002) approach, this study assumes that:

$$D_{x,t} \sim Poisson(E_{x,t}m_{x,t}) \tag{2.10}$$

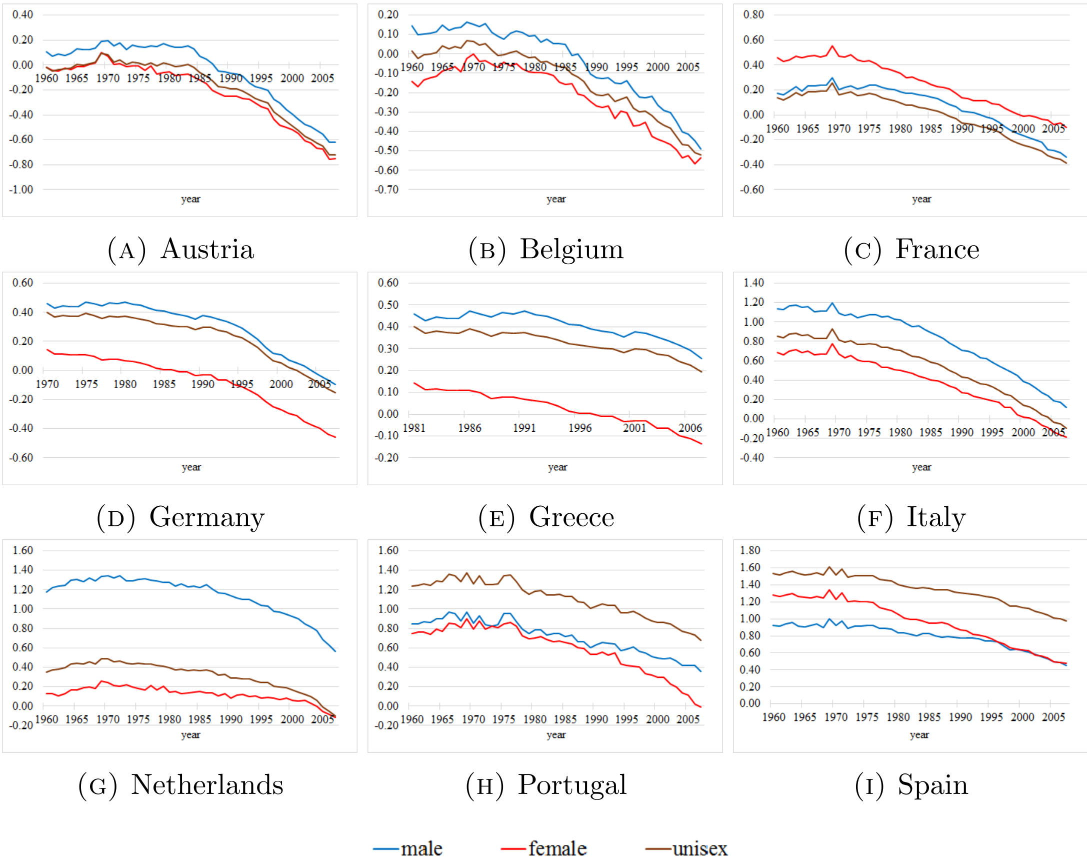
The parameters are then estimated by maximising the log-likelihood function, which is given by

$$L(\psi; D, E) = \sum_{x,t} \{D_{x,t} \ln[E_{x,t}m_{x,t}(\psi)] - E_{x,t}m_{x,t}(\psi) - \ln(D_{x,t}!)\} \tag{2.11}$$

This study uses the R-code from the software package "*Lifemetrics*"[20] to calculate the estimated values of the parameters. This package gives the opportunity to make a good comparison between our proposed model and other existing models. The procedure to fit the model leads to time series estimates of k_t^1, k_t^2, k_t^3, k_t^4,

[20] "Lifemetrics" is an open source toolkit for measuring and managing longevity and mortality risk, designed by J.P. Morgan; full methodology, R code and user guide are available on the website of author: `http://www.macs.hw.ac.uk/~andrewc/lifemetrics/`. This package gives the opportunity to make a good comparison between the proposed model and existing ones.

and γ_{t-x}. After fitting the model, the function takes the fitted values of the time series that suitably fit the ARIMA-processes. For this purpose, this study uses the auto.arima function from the R package forecast, which is fully automatic and returns best ARIMA model according to either Akaike Information Criterion (AIC)[21] or Bayesian Information Criterion (BIC)[22] value.



(A) Austria (B) Belgium (C) France

(D) Germany (E) Greece (F) Italy

(G) Netherlands (H) Portugal (I) Spain

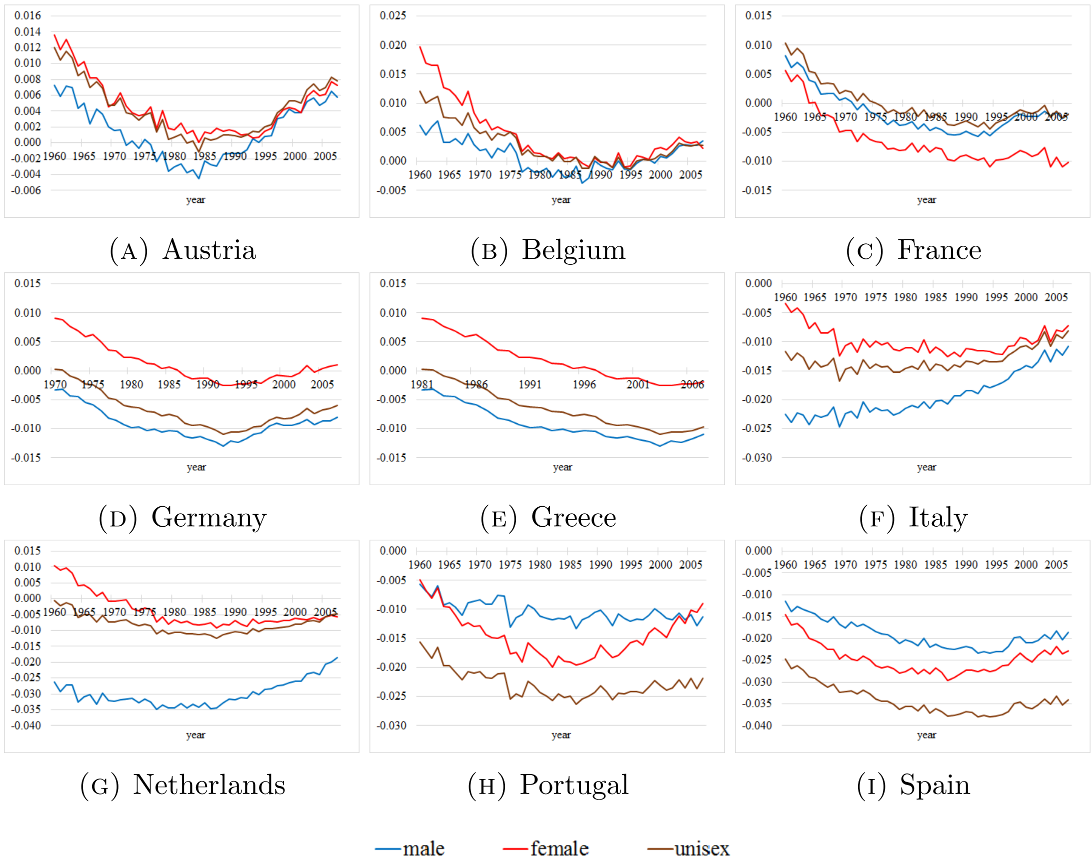
FIGURE 2.4: Plots for time dependent factor k_t^1 for male (blue lines), female (red lines) and unisex (brown lines) data for nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The fitting period is 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007).

Figure 2.4 presents plots of the time dependent factor k_t^1 for male (blue curve), female (red curve) and unisex (brown curve) data for each of the Eurozone countries chosen in this study, Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. Generally, as expected, the k_t^1 factor shows a

[21]Where AIC$= 2k - 2\ln(L)$, L is the maximized value of the likelihood function for the model; k is the number of estimated parameters in the model.

[22]BIC is defined in the following paragraphs of this section.

decreasing and consistent trend for all nine countries over the study period. Contrasting this, the time dependent factor k_t^2 is behaving uniquely for each country with most of them appearing concave upwards (see Figure 2.5). Italy, Portugal and Spain have negative values throughout the period whilst Greece is considered to have a linear decreasing trend of k_t^2.



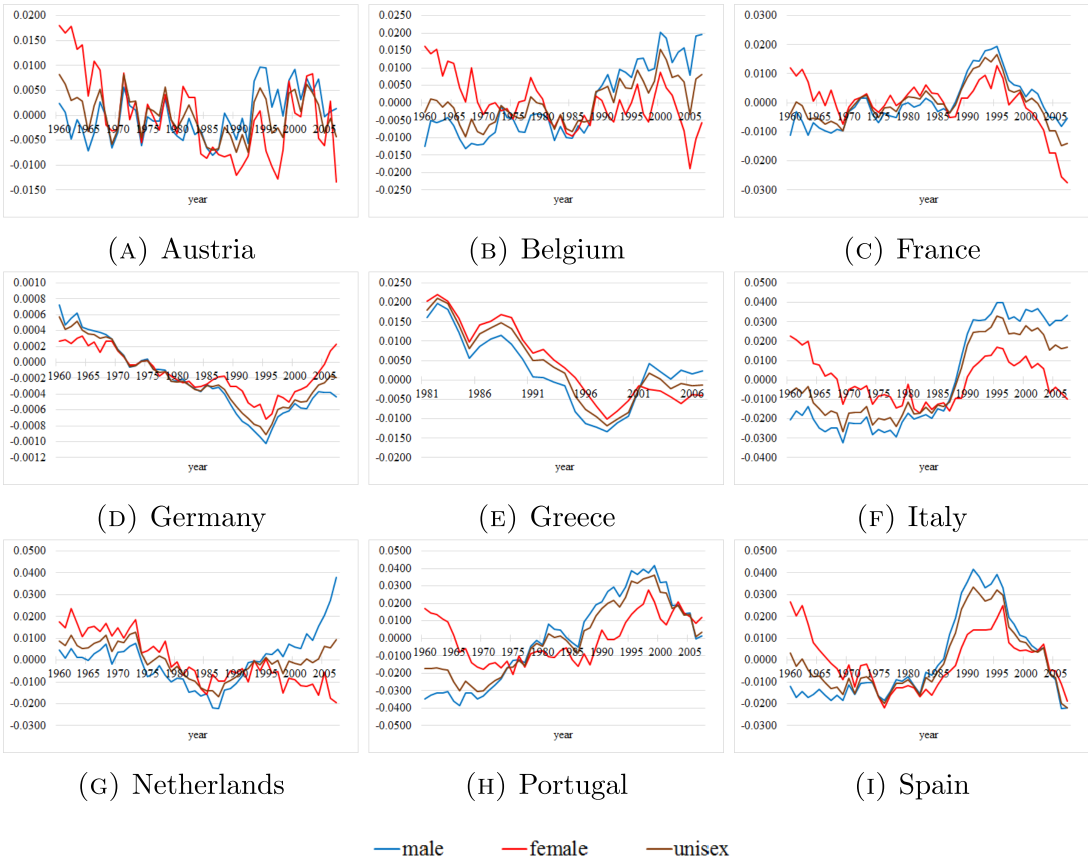
(A) Austria (B) Belgium (C) France

(D) Germany (E) Greece (F) Italy

(G) Netherlands (H) Portugal (I) Spain

FIGURE 2.5: Plots for time dependent factor k_t^2 for male (blue lines), female (red lines) and unisex (brown lines) data for nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The fitting period is 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007).

For time dependent factor of k_t^3 (see Figure 2.6), a consistent trend is observed for each male, female and unisex data (blue, red and brown curves, respectively) for most countries. Moreover, the behaviours of k_t^3 are fluctuating generally across the period for most countries. What is more, this study observes a similar shape of the k_t^3 curves for three groups of countries, i.e., Austria and Belgium; Germany, Greece and the Netherlands; France, Italy, Portugal and Spain.



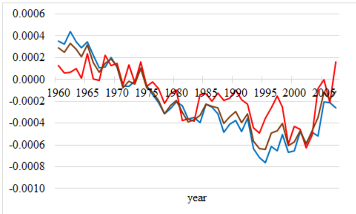
(A) Austria (B) Belgium (C) France

(D) Germany (E) Greece (F) Italy

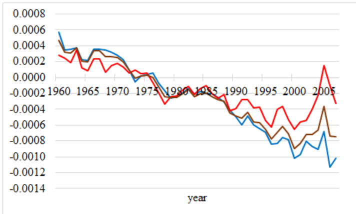
(G) Netherlands (H) Portugal (I) Spain

FIGURE 2.6: Plots for time dependent factor k_t^3 for male (blue lines), female (red lines) and unisex (brown lines) data for nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The fitting period is 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007).

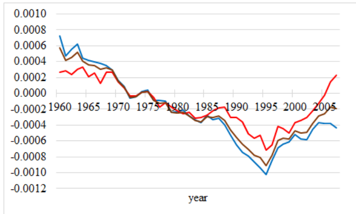
Figure 2.7 presents the time dependent factor k_t^4 for the nine Eurozone countries. Surprisingly, all countries except Greece (as the earliest data available is from 1981) experience a similar trend with positive values during the earlier years and declining values during years 1980–1990. This study also observes a similar shape of curves for the same groups of countries as in the case of k_t^3 (except results for Germany).



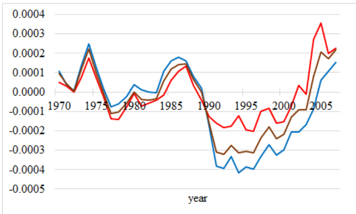
(A) Austria



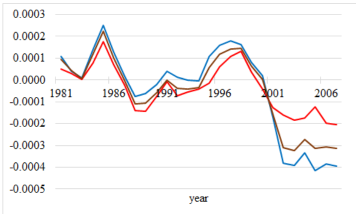
(B) Belgium



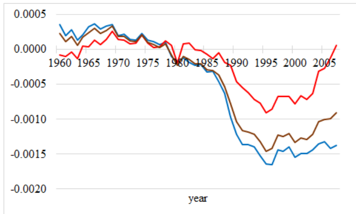
(C) France



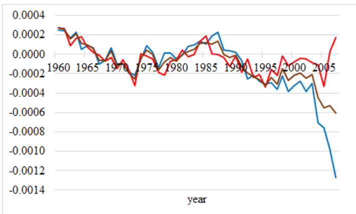
(D) Germany



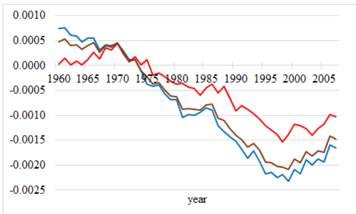
(E) Greece



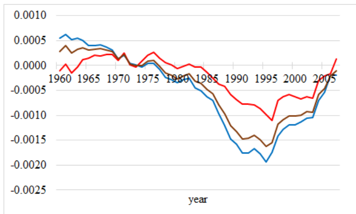
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

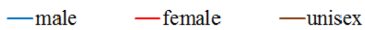


Figure 2.7: Plots for time dependent factor k_t^4 for male (blue lines), female (red lines) and unisex (brown lines) data for nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The fitting period is 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007).

This study then conducts the goodness of fitting by applying the popular measures such as Mean Absolute Percent Error (MAPE), Mean Absolute Deviation (MAD) and Bayesian Information Criterion (BIC). Measures for MAPE and MAD are defined as below:

$$MAPE = \frac{1}{NM} \sum_{x,t} \frac{\|\hat{m}_{x,t} - m_{x,t}\|}{m_{x,t}} \tag{2.12}$$

and

$$MAD = \sum_{x,t} \frac{\|\hat{m}_{x,t} - m_{x,t}\|}{NM} \tag{2.13}$$

where $m_{x,t}$ and $\hat{m}_{x,t}$ are the actual values and estimated values of mortality, M ($M = 90$) is the age dimension and N ($N = 48$) is the time dimension. While the measurement of BIC is as defined by Cairns et al. (2006b):

$$BIC = L(\psi) - \frac{1}{2} kln(p) \tag{2.14}$$

where $L(\psi)$ is the log-likelihood of the estimated parameter ψ, p is the number of observations, and k is the number of parameters being estimated.

Table 2.5 presents the fitting results for male data measured by MAPE, BIC and MAD for the nine Eurozone countries based on the four models: LC, LC–GDP, OL and OL–GDP.

The OL-GDP model provides the best quality fittings of male data for most countries. For instance, in MAPE, the most significant improvement is observable for Italy (32.7%, 28.3% and 14.7% regarding LC, OL and LC-GDP model, respectively), Portugal (17.7%, 13.0% and 5.9% for LC, OL and LC-GDP model, accordingly), Germany (18.6%, 7.5% and 8.9% regarding LC, OL and LC-GDP model, respectively) and France (18.9%, 2.8%, and 3.7% for LC, OL and LC-GDP model, accordingly). While, for Austria, Belgium and the Netherlands, MAPE results for OL-GDP model are slightly worse in comparison to LC-GDP (which has the smallest error).

In terms of MAD, replicating the MAPE results, the best improvement is noticeable for Italy (45.5%, 17.2%, and 32.0% in comparison to LC, OL and LC-GDP model, respectively). What is more, apart from the results for the Netherlands, Portugal and Spain, OL-GDP gives smaller MAD error.

Meanwhile, the best BIC outcomes are recorded for both models of OL and OL-GDP, with Ol-GDP dominating most countries. While for Portugal, LC-GDP is the best.

However, for female data, the results recorded are less positive. As presented in Table 2.6, for both MAPE and MAD measures, most countries are best fitted through the LC-GDP model, except for Greece and the Netherlands. However, the results recorded for BIC vary. Austria, Belgium and the Netherlands are best fitted by the LC model, Portugal and Spain fitted LC-GDP best and France, Germany and Italy are best fitted by the OL-GDP model whilst Greece is best fitted by the OL model. Even the results recorded by the OL-GDP model are not favourable, the variances of its improvement on MAPE are relatively small compared to the LC-GDP model. However, huge improvement is observed on MAD for the LC-GDP model. Contrarily, BIC results provide major improvement of the OL-GDP model compared to the LC model (except for Austria and Belgium).

Table 2.7 presents the results of MAPE, MAD and BIC for unisex data. Basically, the outcomes from the three measures of unisex data are reflecting the outcomes of the male data. In terms of MAPE and MAD results, the best outcomes are recorded by both of the GDP-related models, LC-GDP and OL-GDP. Nonetheless, OL-GDP dominates the best quality fittings of unisex data for most of the countries.

Table 2.5: Quality Fitting Results of MAPE, MAD and BIC for Male

	(LC)	(OL)	(LC-GDP)	Proposed Model (OL-GDP)
Panel A: Mean Absolute Percent Error (MAPE)				
Austria	9.98%	9.66%	**9.07%**	9.55%
Belgium	8.64%	8.41%	**7.99%**	8.19%
France	6.27%	5.22%	5.28%	**5.19%**
Germany*	5.28%	4.64%	4.71%	**4.29%**
Greece**	9.69%	9.45%	9.69%	**9.35%**
Italy	8.07%	7.58%	6.37%	**5.43%**
Netherlands	8.23%	7.58%	**7.03%**	7.19%
Portugal	10.04%	9.50%	8.78%	**8.26%**
Spain	7.45%	7.33%	7.20%	**6.78%**
Panel B: Mean Absolute Deviation (MAD)				
Austria	0.001357	0.001156	0.001191	**0.001137**
Belgium	0.001236	0.000929	0.001047	**0.000882**
France	0.000803	0.000690	0.000750	**0.000682**
Germany*	0.000789	0.000668	0.000719	**0.000650**
Greece**	0.000799	0.000781	0.000799	**0.000702**
Italy	0.001032	0.000680	0.000828	**0.000563**
Netherlands	0.001114	**0.000584**	0.000958	0.000595
Portugal	0.001442	0.001333	**0.001257**	0.001288
Spain	0.000875	0.000855	**0.000767**	0.000769
Panel C: Bayesian Information Criterion (BIC)				
Austria	-20482	**-19498**	-19819	-19696
Belgium	-21443	**-19756**	-20354	-19797
France	-36330	-28844	-32388	**-28793**
Germany*	-29172	-23281	-27829	**-22225**
Greece**	-11205	**-10861**	-11551	-10962
Italy	-43164	-34287	-35274	**-27852**
Netherlands	-22028	-19899	-20447	**-19861**
Portugal	-23873	-23731	**-22520**	-22766
Spain	-34115	-29846	-32797	**-27243**

Notes:* based on the data from 1970 to 2007, ** based on the data from 1981 to 2007.

Table 2.6: Quality Fitting Results of MAPE, MAD and BIC for Female

	(LC)	(OL)	(LC-GDP)	Proposed Model (OL-GDP)
Panel A: Mean Absolute Percent Error (MAPE)				
Austria	14.30%	14.51%	**14.19%**	14.44%
Belgium	10.61%	10.84%	**10.58%**	10.80%
France	6.10%	6.01%	**5.38%**	5.70%
Germany*	5.06%	5.36%	**4.95%**	5.11%
Greece**	15.77%	**14.34%**	15.78%	14.50%
Italy	7.01%	6.93%	**6.11%**	6.55%
Netherlands	**8.52%**	9.09%	**8.52%**	8.66%
Portugal	9.90%	11.37%	**9.43%**	10.82%
Spain	7.71%	8.61%	**7.10%**	8.18%
Panel B: Mean Absolute Deviation (MAD)				
Austria	0.000755	0.000803	**0.000729**	0.000801
Belgium	0.000622	0.000751	**0.000618**	0.000764
France	0.000438	0.000510	**0.000383**	0.000514
Germany*	0.000367	0.000471	**0.000365**	0.000483
Greece**	0.000849	0.000681	0.000849	**0.000631**
Italy	0.000567	0.000567	**0.000448**	0.000531
Netherlands	**0.000456**	0.000610	**0.000456**	0.000606
Portugal	0.000955	0.000931	**0.000806**	0.000907
Spain	0.000621	0.000766	**0.000463**	0.000745
Panel C: Bayesian Information Criterion (BIC)				
Austria	**-18165**	-18331	-18424	-18483
Belgium	**-18652**	-18731	-18999	-18770
France	-27149	-25310	-25987	**-24951**
Germany*	-24863	-21823	-24976	**-21153**
Greece**	-10944	**-10188**	-11291	-10289
Italy	-29027	-26674	-26998	**-25835**
Netherlands	**-18320**	-18943	-18690	-18876
Portugal	-20559	-21024	**-19878**	-20919
Spain	-26140	-26073	**-24407**	-25310

Notes:* based on the data from 1970 to 2007, ** based on the data from 1981 to 2007.

Table 2.7: Quality Fitting Results of MAPE, MAD and BIC for Unisex

	(LC)	(OL)	(LC-GDP)	Proposed Model (OL-GDP)
Panel A: Mean Absolute Percent Error (MAPE)				
Austria	8.09%	7.99%	**7.42**%	7.97%
Belgium	6.72%	6.65%	**6.46**%	6.51%
France	5.62%	4.73%	4.84%	**4.51**%
Germany*	4.68%	4.22%	4.49%	**3.88**%
Greece**	8.06%	7.66%	8.06%	**7.53**%
Italy	6.78%	6.43%	5.55%	**4.94**%
Netherlands	5.67%	6.06%	**5.32**%	5.76%
Portugal	8.32%	8.76%	**7.45**%	7.55%
Spain	6.55%	7.00%	**6.20**%	6.47%
Panel B: Mean Absolute Deviation (MAD)				
Austria	0.000822	0.000757	**0.000745**	0.000751
Belgium	0.000667	0.000618	0.000625	**0.000601**
France	0.000511	0.000483	0.000492	**0.000472**
Germany*	0.000528	0.000469	0.000521	**0.000461**
Greece**	0.000694	0.000590	0.000694	**0.000587**
Italy	0.000649	0.000548	0.000556	**0.000456**
Netherlands	0.000700	0.000454	0.000688	**0.000432**
Portugal	0.000996	0.000959	**0.000822**	0.000891
Spain	0.000653	0.000759	**0.000505**	0.000707
Panel C: Bayesian Information Criterion (BIC)				
Austria	**-22450**	-21367	-21793	-21458
Belgium	**-22382**	-20977	-21998	-21037
France	-41057	-32222	-37648	**-31335**
Germany*	-39747	-28126	-38958	**-26029**
Greece**	-12523	**-11622**	-12870	-11698
Italy	-47870	-37899	-40406	**-32044**
Netherlands	-22293	-21296	-22075	**-21207**
Portugal	-26846	-27212	**-24845**	-26063
Spain	-39148	-35698	-36469	**-32325**

Notes:* based on the data from 1970 to 2007, ** based on the data from 1981 to 2007.

Generally, the proposed modification of the OL model (OL-GDP) gives better results for most countries in comparison to other models. As presented in Table 2.5, the proposed model (OL-GDP) performed well for most of the countries for male data. For MAPE, six out of nine countries are best fitted by the OL-GDP model. Likewise, for MAD, six out of nine countries are best fitted by the proposed model, OL-GDP, and for BIC, five out of nine countries are best fitted by the OL-GDP.

On the other hand, for female data (see Table 2.6), LC-GDP dominates to best fit most countries for MAPE and MAD measures. Contrasting this, for BIC measure, OL-GDP and LC models outperform other models by fitting most of the countries. France, Germany and Italy fit OL-GDP best whilst Austria, Belgium and the Netherlands fit LC model well. On another hand, LC-GDP fits Portugal and Spain better and OL fits Greece well.

It is interesting to note that although the best fitting of female data does not promote the new proposed model, OL-GDP, the results still stand undoubtedly, supporting the basis that the economic factor does give a significant impact on the mortality trends. Factually, the next GDP incorporated factor of mortality model, i.e., LC-GDP provides the best fitting of MAPE and MAD for all female data in the selected Eurozone countries. Thus, these findings contribute to existing literature such as Bhargava et al. (2001), Hanewald (2011), French and O'Hare (2014), Niu and Melenberg (2014) and Khemka and Roberts (2015) in promoting the significant impact of economic growth on mortality.

For unisex, as presented in Table 2.7, the proposed modification of the OL model (OL-GDP) gives better results for most countries in comparison to the other models. The OL-GDP model dominates for most of the countries. For MAPE, LC-GDP outperforms the OL-GDP for one country, but for MAD, six out of nine countries are best fitted by the proposed model, OL-GDP, and for BIC, five out of nine countries are best fitted by the OL-GDP.

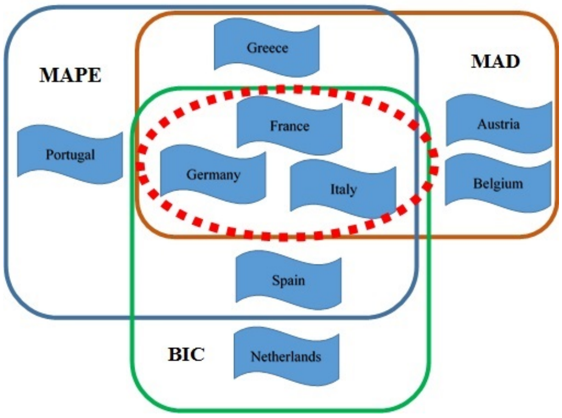
The OL-GDP model gives better results for all MAPE, MAD and BIC measures, for both male and unisex for France, Germany and Italy. However, for female data, LC-GDP provides better results for MAPE and MAD. For male, if one only considers the MAPE measure, Greece, Portugal and Spain are also on the list of better fitting. MAD results are fitted best for almost all countries except for the Netherlands, Portugal and Spain. For BIC measure, better fittings are observed for France, Germany, Italy, the Netherlands and Spain. For unisex, MAPE fits best for Austria, France, Germany, Greece and Italy. Whilst MAD fits Belgium, France, Germany, Greece and Italy well, and BIC works best for France, Germany, Italy, the Netherlands and Spain. For better understanding, this study maps the summary of quality fittings for all countries in Figure 2.8.

As presented in Figure 2.8, France, Germany and Italy provide the best fittings to the new proposed model, OL-GDP for each MAPE, MAD and BIC results of male, female and unisex. This might not be surprising as these three countries are the most populous among the nine. As reported by Chen et al. (2017), the size of a population has a significant effect on the uncertainty about the estimated parameters and mortality projections. In their study, they found that there exists a bias in the estimated covariance matrix of the random walk fitted to the period effects when the size of the underlying population is small. As a consequence, prediction intervals are rather wide for small populations even when parameter uncertainty is ignored.

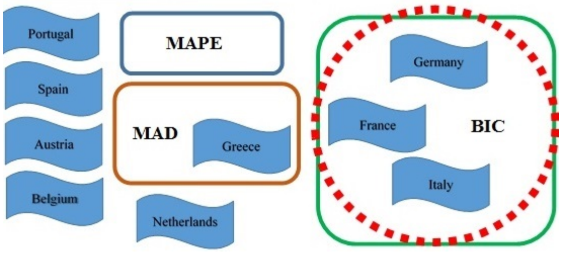
This study first analyses the behaviour of France towards the sensitivity of its mortality rates on logarithm GDP for each year of age for male, female and unisex. The trend of correlations of average mortality and logarithm GDP (over time) across ages, are quite similar for male, female and unisex. However, the correlations of logarithm GDP and mortality rates are relatively lower during age 15–30 years old especially for males, followed by unisex and females. Apparently, the mortality rates at middle ages (50 years old onwards) are highly correlated with economic growth. This might be generally due to the impact of the economy towards the wellbeing of the mid-age populations.

For Germany, similar trends of correlations were observed among male, female and unisex. They are relatively correlated throughout the period with female remaining the most negatively correlated as compared to unisex and male. In comparison to France, Germany appears to be less correlated. Unlike France, the age group between 30–40 years old of Germany tends to be less correlated as compared to other ages, especially for male.

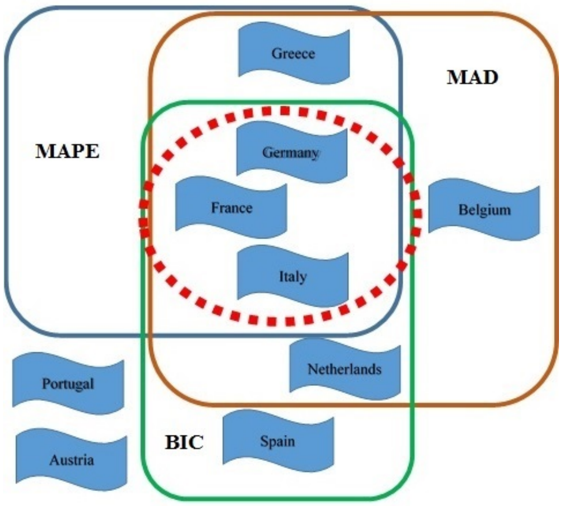
As for Italy, similar patterns were also observed among male, female and unisex for its correlations between mortality rates and GDP trend. As presented in Figure 10, some odd trend for male was captured at young ages between 10–20 years old. The trend of female is likely to be consistent throughout the ages with very high negative correlations between the GDP and mortality trends. The age group



(A) Male



(B) Female



(C) Unisex

FIGURE 2.8: Intersections of France, Germany and Italy for the OL-GDP model. (a) Male, (b) Female, (c) Unisex.

between 20–40 years old highlights a weaker correlation with male being least correlated.

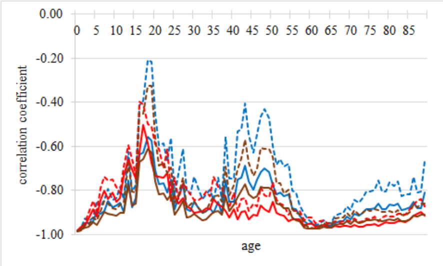
Overall, this study observes that the middle age group between 15–40 years old are less sensitive and unlikely correlated to the economic growth. It seems that at this group of age, the decreasing trend in mortality rates does not relatively translate to the increasing trend in GDP. It appears to us that at this age group, most of the people are having an excellent condition of their health generally. Hence, the economic status does not really give an impact on their health status.

2.4.3 Robustness Checks

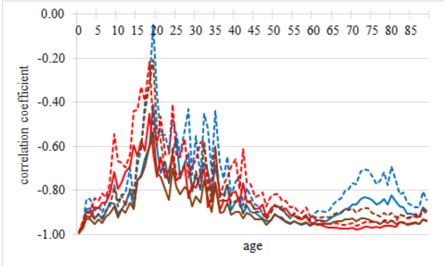
Robustness testing is a very important feature of verifying a good model. In this subsection, this study investigates the robustness of correlation between mortality rates by age and GDP changes over time. Moreover, this study also analyses the robustness of the results for the proposed model. Figure 2.9 plots separate Pearson's correlation coefficient for male, female and unisex (blue, red and brown curves accordingly) data of the nine countries for two periods: 1960–2007 (solid lines) and 1960–1997 (dashed lines) with appropriate modifications for Germany (1970–2007, 1970–1997) and Greece (1981–2007, 1981–1997).

Generally, this study observes similar trends with significant negative correlations for all countries and genders for both periods of time (38 and 48 years respectively, except for Germany, 28 and 38 years and Greece, 17 and 27 years). However, if this study looks into each country specifically, every country displays different trends of correlations. Austria, Belgium and Greece have quite similar trends for all genders in both periods. However, only Greece experiences a significant positive correlation (up to 0.8) for male and unisex for ages 20–50. Portugal and Spain experience a positive correlation for male between 0.3 and 0.4 during ages 15–30. For the most part, the fluctuations of correlations can be seen clearly for ages 20–40 for all countries. Furthermore, this study found that a shorter period of observations results in lower correlations. This can be clearly observed for all countries except Greece. Apparently, the effect of the GDP on mortality rates is more meaningful in the longer windows as opposed to shorter windows.

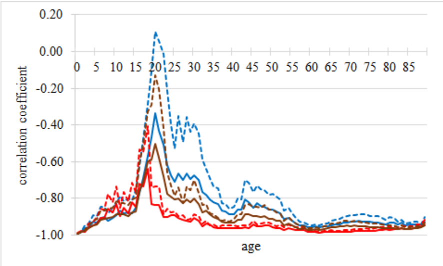
It is also interesting to note that regardless of the number of observations fitted during the process, the general shape of the correlations remains the same. This indicates that an age specific GDP-related factor is not very sensitive to the range of data used during the parameter fitting process.



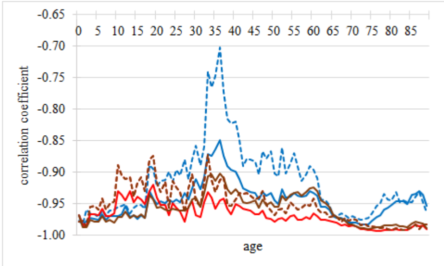
(A) cg_x for Austria



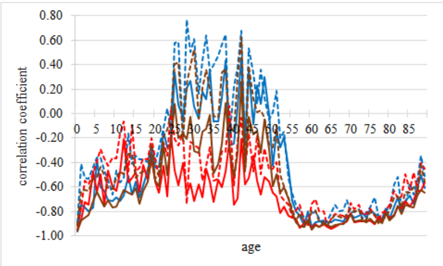
(B) cg_x for Belgium



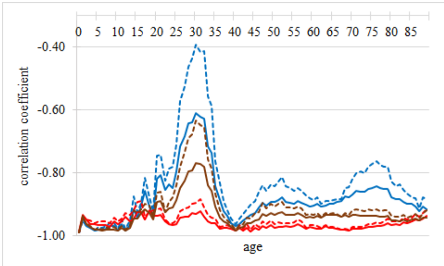
(C) cg_x for France



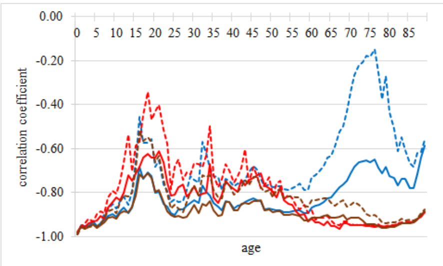
(D) cg_x for Germany



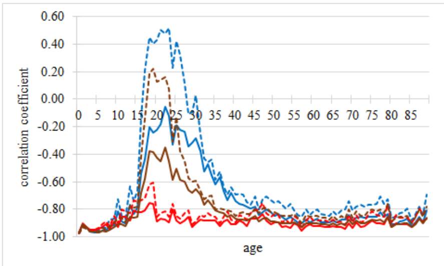
(E) cg_x for Greece



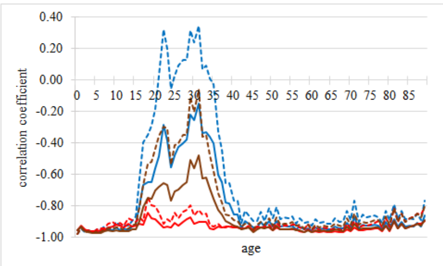
(F) cg_x for Italy



(G) cg_x for Netherlands



(H) cg_x for Portugal



(I) cg_x for Spain

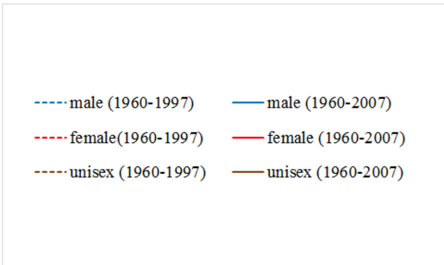


FIGURE 2.9: Correlation Coefficients between GDP and Mortality Rates for Male (blue), Female (red) and Unisex (brown) Data from Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain for two periods: 1960–2007 (solid lines) and 1960–1997 (dashed lines) with appropriate modifications for Germany (1970–2007, 1970–1997) and Greece (1981–2007, 1981–1997).

During the fitting process, this study uses a dataset over the period from 1960 to 2007 (with appropriate modifications for Germany and Greece), and for the age range of 0–89. A few years of the dataset are left for comparison purposes. This study reduces the fitting period from 1960–2007 to 1960–1997 in order to have a longer comparison period for the forecasting results. MAPE results for shorter periods of time are presented in Table 2.8.

Table 2.8: MAPE for each of Eurozone Country over Fitting Period 1960–1997 (except Germany and Greece where this study has 1970–1997 and 1981–1997, respectively).

	male	**female**	**unisex**
Austria	8.62%	12.48%	6.97%
Belgium	7.39%	9.92%	5.95%
France	5.05%	5.17%	4.37%
Germany	3.42%	3.91%	2.81%
Greece	8.39%	10.92%	6.54%
Italy	5.10%	5.50%	4.48%
Netherlands	6.52%	7.59%	5.19%
Portugal	7.42%	9.70%	6.80%
Spain	6.49%	7.39%	6.13%

2.5 Forecasting Life Expectancy and Longevity Risk

This study uses the OL-GDP model to forecast future mortality rates for each Eurozone country by fitting their historical mortality rates, respectively. The forecasting caters for each of male, female and unisex ages 0–89 over the period of 2008–2027. Specifically, this study groups the data into two sections, in-sample and out-of-sample data for MAPE. The in-sample data used for Austria, Belgium, France, Italy, the Netherlands, Portugal and Spain are from 1960 to 2007, whilst for Germany and Greece are 1970–2007 and 1981–2007, respectively. This study uses the remaining of 2008–2012 data to test the forecasting performance (out-of-sample) by using MAPE.

Time-dependent factors of the OL-GDP model are forecasted using the Autoregressive Moving Average (ARIMA) and Exponential Smoothing (ES) for the period from 2008–2027. For the ARIMA approach, this study uses the forecast.Arima function from R package forecast. As the data set used differs for each country and each gender, parameters estimates for the ARIMA (p, d, q)[23] also need to be tailored to each country and gender respectively. For instance, Table 2.9 presents the parameters (p, d, q) used for each ARIMA model in each Eurozone country of k_t^1 factor for male, female and unisex data.

TABLE 2.9: ARIMA Model Parameters (p, d, q) for the k_t^1 Factor for each of Eurozone Country.

	male	**female**	**unisex**
Austria	$(0, 2, 1)$	$(0, 2, 1)$	$(0, 2, 1)$
Belgium	$(0, 2, 1)$	$(0, 2, 2)$	$(0, 2, 1)$
France	$(2, 2, 1)$	$(0, 1, 1)$	$(2, 2, 1)$
Germany	$(1, 2, 0)$	$(1, 2, 0)$	$(1, 2, 0)$
Greece	$(0, 1, 1)$	$(0, 1, 1)$	$(0, 1, 1)$
Italy	$(0, 2, 2)$	$(0, 2, 2)$	$(2, 2, 1)$
Netherlands	$(1, 2, 1)$	$(1, 2, 1)$	$(2, 2, 0)$
Portugal	$(0, 1, 1)$	$(1, 2, 1)$	$(1, 1, 0)$
Spain	$(1, 2, 1)$	$(0, 2, 2)$	$(1, 2, 1)$

For comparison purposes, this study estimates the same dataset using the ES technique[24] (for more details, see Hyndman and Booth (2008)) through the *ets* function from the *R* package forecast. Additionally, to validate the ability of the model in forecasting better results, this study extends the forecast on to other models under study, LC, OL and LC-GDP.

[23]Parameters p, d, and q are non-negative integers, p is the order of the autoregressive model, d is the degree of differencing, and q is the order of the moving-average model.

[24]Exponential Smoothing assigns exponentially decreasing weights as the observation get older. In other words, recent observations are given relatively more weight in forecasting than the older observations.

As mentioned earlier, this study adopts the Mean Absolute Percentage Error (MAPE) to study the performance of in-sample fitting and out-of-sample forecasting. For some reason, MAPE performs better than MSE as the mortality rate varies significantly over different ages (Luo et al. (2016)). MSE will be influenced solely by the larger mortality rate as varies drastically across age groups. Contrary, MAPE uses the percentage error that can better assess the in-sample fitting and out-of-sample forecasting.

Table 2.10 illustrates the 5-year out-of-sample test for MAPE of the LC, OL, LC-GDP and OL-GDP models for the period 2008–2012, using ARIMA and ES estimation (Panel A and B, respectively). Overall, the proposed model, OL-GDP gives better results in comparison to the LC, OL and LC-GDP models for all countries and genders under study. Particularly, sizes of the improvement depend on the respective country, gender and methodology. For example, for male's results, an improvement of 42% is observed for Italy under ARIMA, in comparison to the next best model, LC-GDP (58% against LC-GDP for ES). Similar performance is observed for unisex data on Italy (41%, 46% against the next best model, LC for ARIMA and ES, respectively). While, for female data, France scored the highest rates of improvement, 14% for ARIMA and 17% for ES against the next best model, LC-GDP. On average, ES performs better than ARIMA by providing higher improvement rates against the next best model, LC-GDP for both male and female, and OL for unisex. It is interesting to note that, for countries and genders that promoting other models, our proposed model, OL-GDP stands to be the next best model of choice compared to others (Male: Austria and Spain for both ARIMA and ES, Female: Spain for ARIMA only, and Unisex: Austria and Spain for both ARIMA and ES and Portugal for ARIMA only). Only for female Austria (both ARIMA and ES) and unisex Portugal (ES) we ranked third.

TABLE 2.10: Out-of-sample Test Results for MAPE (over the period 2008–2012).

	male				**female**				**unisex**			
	LC	OL	LC-GDP	OL-GDP	LC	OL	LC-GDP	OL-GDP	LC	OL	LC-GDP	OL-GDP
Panel A: ARIMA												
Austria	20.23%	21.89%	**17.08%**	19.06%	20.46%	19.22%	**18.25%**	20.25%	16.86%	**15.03%**	17.07%	16.55%
Belgium	18.09%	18.00%	16.96%	**13.63%**	19.39%	16.73%	19.33%	**16.47%**	14.57%	10.77%	14.12%	**9.92%**
France	13.37%	7.77%	10.89%	**7.64%**	13.96%	11.73%	10.55%	**9.07%**	13.03%	12.83%	12.86%	**12.67%**
Germany	11.53%	9.67%	10.23%	**9.61%**	12.61%	12.67%	12.92%	**12.37%**	9.13%	8.84%	8.45%	**8.26%**
Greece	15.21%	16.04%	15.20%	**15.16%**	18.39%	25.75%	18.39%	**16.56%**	11.95%	11.58%	11.95%	**10.54%**
Italy	19.21%	27.38%	19.03%	**11.04%**	15.60%	16.82%	15.69%	**15.12%**	14.61%	18.28%	16.88%	**8.58%**
Netherlands	21.41%	14.43%	19.32%	**13.89%**	17.92%	15.55%	15.18%	**15.09%**	13.15%	12.57%	14.26%	**10.97%**
Portugal	38.13%	18.31%	20.75%	**18.29%**	21.20%	22.82%	21.16%	**20.80%**	29.51%	**13.23%**	17.38%	14.52%
Spain	37.09%	**13.85%**	33.95%	27.61%	17.28%	**13.68%**	20.79%	16.93%	28.65%	**12.73%**	31.39%	20.85%
Panel B: Exponential Smoothing												
Austria	18.20%	21.63%	**17.77%**	18.11%	20.59%	19.01%	**18.25%**	20.43%	16.87%	**15.25%**	17.12%	15.71%
Belgium	18.08%	18.28%	16.94%	**13.67%**	19.12%	16.14%	19.06%	**15.61%**	14.43%	10.02%	13.80%	**9.89%**
France	13.36%	7.78%	10.86%	**7.34%**	13.96%	11.13%	10.56%	**8.77%**	12.66%	7.79%	11.92%	**6.27%**
Germany	11.54%	9.59%	10.32%	**8.85%**	9.73%	10.84%	10.62%	**9.35%**	9.33%	9.00%	8.64%	**7.64%**
Greece	15.18%	16.77%	15.17%	**14.28%**	18.35%	25.19%	18.34%	**17.44%**	11.89%	14.13%	11.89%	**11.21%**
Italy	19.23%	27.39%	19.15%	**8.06%**	15.39%	17.52%	15.30%	**14.38%**	14.57%	17.92%	16.86%	**7.81%**
Netherlands	22.10%	19.80%	19.45%	**18.46%**	16.64%	16.02%	15.90%	**15.50%**	13.42%	11.95%	14.60%	**10.83%**
Portugal	37.33%	20.97%	21.03%	**20.00%**	20.48%	26.68%	21.60%	**20.24%**	28.20%	16.43%	17.81%	**18.38%**
Spain	34.66%	**13.91%**	30.72%	25.98%	17.52%	13.57%	21.00%	**13.42%**	29.29%	**13.28%**	30.55%	20.27%

In view of a shorter period of forecasting (5 years) previously, this study considers a longer forecasting period as an additional robustness check. This study uses the 38-year dataset (from 1960 to 1997)[25] to calculate factors from the models, for having a longer forecasting period of 15 years (1998–2012).

TABLE 2.11: Out-of-sample Test Results for MAPE (over the period 1998–2012).

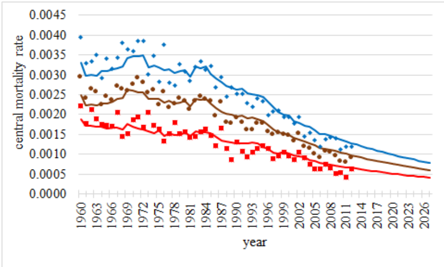
	male		**female**		**unisex**	
	LC	OL-GDP	LC	OL-GDP	LC	OL-GDP
Panel A: ARIMA						
Austria	**19.60%**	19.75%	**25.99%**	28.59%	16.10%	**13.94%**
Belgium	18.12%	**16.75%**	19.32%	**15.55%**	15.28%	**12.89%**
France	23.56%	**12.23%**	17.61%	**10.79%**	19.79%	**8.34%**
Germany	15.94%	**11.31%**	16.32%	**14.67%**	15.33%	**15.28%**
Greece	19.59%	**17.75%**	**30.56%**	40.61%	15.35%	**12.17%**
Italy	21.03%	**12.14%**	16.01%	**15.81%**	15.26%	**10.79%**
Netherlands	19.53%	**16.20%**	20.43%	**18.88%**	15.93%	**14.01%**
Portugal	42.08%	**27.38%**	21.28%	**19.71%**	31.49%	**27.38%**
Spain	34.83%	**31.98%**	18.95%	**18.78%**	24.41%	**17.59%**
Panel B: Exponential Smoothing						
Austria	**19.73%**	22.83%	**25.99%**	26.31%	16.09%	**15.90%**
Belgium	18.18%	**14.41%**	19.01%	**15.34%**	14.57%	**12.30%**
France	20.53%	**12.72%**	17.30%	**17.22%**	17.88%	**12.55%**
Germany	14.47%	**12.16%**	10.40%	**10.37%**	12.39%	**9.97%**
Greece	19.57%	**19.17%**	**32.65%**	33.64%	15.39%	**15.94%**
Italy	20.97%	**13.45%**	16.10%	**15.43%**	15.25%	**11.00%**
Netherlands	19.47%	**19.32%**	26.30%	**26.02%**	16.13%	**11.18%**
Portugal	40.78%	**33.43%**	19.90%	**19.27%**	30.32%	**29.17%**
Spain	34.78%	**22.62%**	15.62%	**14.47%**	24.38%	**17.77%**

[25]Except for Germany and Greece for which data is available from 1970 and 1981, respectively.

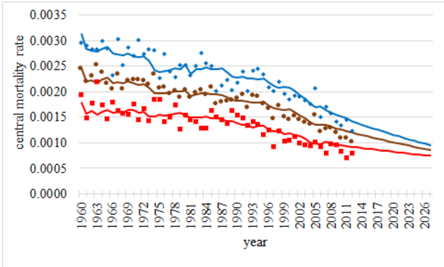
Table 2.11 illustrates the 15-year out-of-sample test for MAPE of the proposed new model and the LC model over the period of 1998–2012, using ARIMA and ES estimation (Panel A and B, respectively). As presented in the table, the forecasting performance varies between countries, genders, and methods used. Undoubtedly, the proposed model fits better than the LC Model for most countries and genders except for Austria (male-female, ARIMA-ES) and Greece (female, ARIMA-ES and unisex, ES). For this period, this study also observes a better improvement of the LC model for forecasting results under the ARIMA approach (the most significant is France, 48.1%, 38.7% and 57.9% for male, female and unisex data, respectively). While for ES, better results are observed for males from Austria, Belgium and Spain (15.7%, 20.8%, 35.0%, respectively), and for unisex data from Germany and the Netherlands (19.5%, 30.7%, respectively).

In managing the uncertainty of forecasting, this study forecasts the mortality rates based on intervals of 5 and 95%percentiles. These intervals are also applicable to all k_t estimations that are being projected from the proposed model. This study presents the illustrations of the forecasting results of mortality rates based on the proposed model in Figure 2.10. The mortality rates are fitted from 1960 to 2007 (except for Germany, 1970 and Greece, 1981) and forecasted from 2008 to 2027, demonstrating the forecasting results for Male age 40 for each Eurozone country (see Figure 2.10). Except for Greece, all other countries display sensible forecasting results with a decreasing trend of mortality rates. The forecast results for Greece exhibit greater volatility might be due to the country's debt crisis post 2009. The behaviour of k_t is depending on its volatility (Niu and Melenberg, 2014). As shown in Figure 2.10, the estimated Male k_t is least volatile in Italy and Spain, corresponding to the narrowest intervals of the forecasts that are represented by the dashed lines. Meanwhile, the estimated Male k_t is most volatile in Greece, referring to the widest intervals of the forecast.

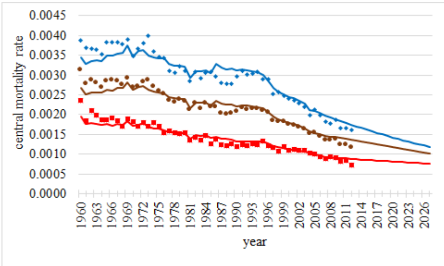
At the end of this section, this study presents estimation examples from the OL-GDP model. Figure 2.10 illustrates the fitted mortality rates for male (blue curve), female (red curve) and unisex (brown curve) at age 40, from 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007) followed by forecasting results from 2008–2027 (using ARIMA process). Additionally, the HMD data is marked for comparison purposes (data from 1960 to 2012, except Germany: 1970–2012, and Greece: 1981–2012). In general, shapes of the fitted mortality curve vary between countries and genders, however, this study observes a decreasing trend for all of countries. Similar patterns are observable for the predicted values. Only for Greece, mortality curve for male is increasing. This peculiar trend may result from the economic crisis that hit Greece the hardest in 2008. A study conducted by Laliotis et al. (2016), exploring the causal effect of the trend of mortality rates due to the financial crisis that hit Europe, in particular Greece in 2008, they found that, the mortality rates for Greece continued to decline after the onset of the crisis period, but at a slower pace than before the recession. This reduction in decline is more evident for females than for males. The reasons for this might be due to sharp increase of death from suicides, homicides, mental health problems, nervous system diseases and digestive diseases. As such, most of the causes of death mentioned are related closely to the post crisis effect. Moreover, these causes of death are contributed largely by males rather than females, as men tend to be the breadwinner for the families.



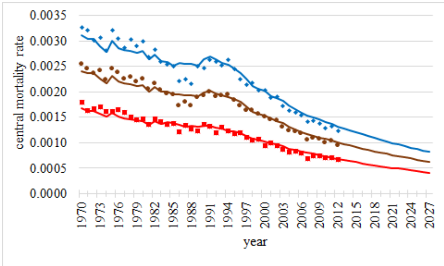
(A) $m_{t,x}$ for Austria



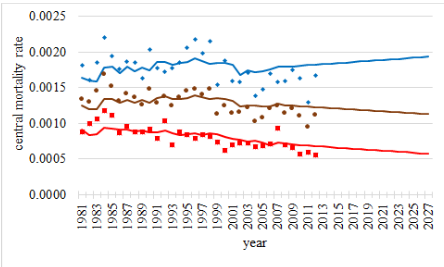
(B) $m_{t,x}$ for Belgium



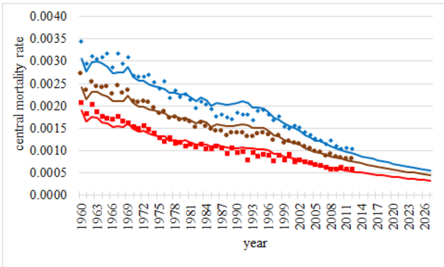
(C) $m_{t,x}$ for France



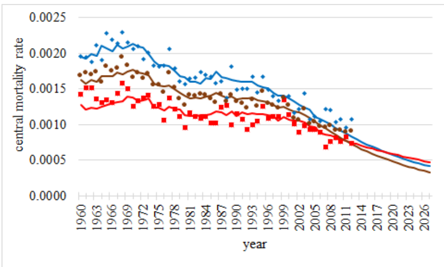
(D) $m_{t,x}$ for Germany



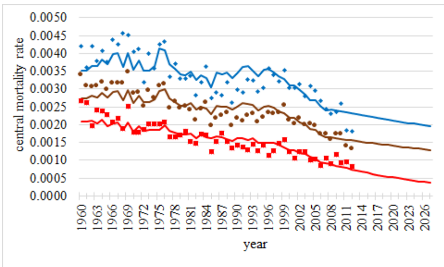
(E) $m_{t,x}$ for Greece



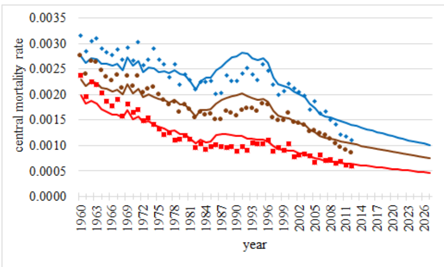
(F) $m_{t,x}$ for Italy



(G) $m_{t,x}$ for Netherlands



(H) $m_{t,x}$ for Portugal



(I) $m_{t,x}$ for Spain

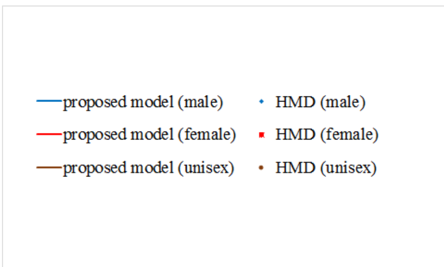


FIGURE 2.10: Mortality Rates for 40 year old Males (blue), Females (red) and Unisex (brown) Data from Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. Fitted to the HMD data from 1960 to 2007 (except Germany: 1970–2007, and Greece: 1981–2007) and followed by forecasting results 2008–2027 (solid lines). Markers show relevant data from HMD.

2.6 Conclusion

The recent study conducted by Niu and Melenberg (2014) provides motivation to further explore the association between the trends in mortality and GDP growth. Unlike Niu and Melenberg (2014) who based their investigation on the Lee and Carter (1992) (LC) model, this study is based on the O'Hare and Li (2012) (OL) model, as this model captures the non-linear profile of mortality at a young age as well as providing a better fit for a range of countries worldwide. The former study incorporated the presence of GDP growth directly in the LC model (LC-GDP) for six OECD countries (the USA, Canada, the UK, the Netherlands, Australia and Japan). Differently from Niu and Melenberg (2014), this study focuses on selected Eurozone countires,(Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain) and proposes a new model, (OL-GDP) by including the GDP-age dependent factor into the existing model of OL. This association reflects the GDP impact on the respective age individually.

To summarize: First, this study investigates the correlations between the latent mortality index of k_t^1 and the growth of logarithmic GDP based on the LC model. The observations provide strong negative correlations between average mortality rates and GDP trend for all sexes across all ages. Moreover, correlations at older age are much stronger for most Eurozone countries except Greece. Evidently, through the quality of fitting measures of MAPE, MAD and BIC, the new model, OL-GDP fits best most countries as compared to other mortality models (LC, OL and LC-GDP). This study notes the impact of GDP is more significant especially in the most populous countries as the best fittings of the OL-GDP model are observed for France, Germany and Italy. For this reason, this model may also appropriate to be extended to other developed country like US and also to other developing countries like China and India since there are populous countries.

What is more, OL-GDP model also demonstrates better quality forecasts of mortality rates through the applications of ARIMA process and Exponential Smoothing techniques. Essentially, the MAPE measure of forecasting shows a significant improvement in forecasting results for shorter (5 years) and longer (15 years) periods of times in comparison to the LC model which acts as the reference model. The impact of economic growth on mortality dynamics has proved to be noteworthy. Further investigation on this will certainly shed some light on mortality studies. An in-depth study on this subject will absolutely give a clearer picture of this association in explaining the mortality trends more precisely. At the same time, this will definitely improve the mortality forecasting accuracy.

Chapter 3

OL-GDP Model Impacts on Insurance Pricing and Reserving

3.1 Introduction

Over the past decades, the world has experienced rapid improvements in life expectancies. Many industrialised countries witnessed a consistent rising of human longevity at a promising and significant pace. The main reason driving this phenomenon is said to be a reduction of death rates among the older people (Cox et al., 2010; Denuit et al., 2007). Factors like socio-economic status, medical innovations, nutrition habits and environmental conditions are believed to be the contributing aspects of this occurrence (French and O'Hare, 2014; Gaille and Sherris, 2010; Post and Hanewald, 2013). Whilst reflecting a positive milestone of society's social standing and well-being, longevity has also posed challenges and threats for some organisations.

The evolution of longevity trends is believed to provide a new curve on the standard life expectancy table, underestimating the current one. This threat has drawn the attention of organisations like pension funds, life and health insurers and also individuals. In particular, holders of longevity risk need to be more mindful in terms of funding, pricing and reserving their products sufficiently. Governments need to be vigilant in funding the ageing people, insurers need to be attentive in providing conducive and inclusive healthcare solutions, and individuals need to be careful in planning their favourable post-retirement. Consequently, a suitable choice of mortality models and a reliable reference of forecasted mortality rates are deemed crucial in mitigating the longevity risk. Therefore, an accurate forecast of future mortality experiences should be done carefully and precisely.

Apparently, the relationship between economic growth and mortality improvement has long been discussed and presented by many researchers since the beginning of the 21st century (Regidor et al., 2016; Niu and Melenberg, 2014; French and O'Hare, 2014; Hanewald, 2011; Granados, 2008; Preston, 2007; Brenner, 2005; Bhargava et al., 2001). They have mainly focused on the effect of macroeconomic fluctuations on mortality in different socio-economic groups of various countries. However, only a small number of researchers extended this study into the establishment of a new mortality model, incorporating the effect of the economic fluctuations or GDP (Boonen and Li, 2017; French and O'Hare, 2014; Niu and Melenberg, 2014).

Earlier (in Subsection 2.4.1), this study had established a new mortality model describing the GDP-age dependent factor and its connection to the mortality experience (please refer to Equation 2.7). Due to its relevance and reliability, this model is well-suited and best-fitted for most of the selected Eurozone countries under study. The Model also provides more plausible forecasts of mortality rates over the 20-year projections for a significant number of Eurozone countries.

One of the main objectives of this study is to investigate the robustness of the OL-GDP model. As presented earlier, the correlation coefficients between GDP and mortality, cg_x, is used in formulating the OL-GDP model. Thus, this model may suffer from sampling error in estimating the parameter of cg_x. However, upon fitting the data, the OL-GDP model still outperform other mortality models by providing the least error compared to other models. Hence, to address the sampling error may not significantly improve the model.

Besides, this study examines the impact of the OL-GDP model on its ability to forecast the mortality rates if the parameters involved in the model are modified. Further analysis would also be conducted to study sensitivity of each parameter's variations against the model. Subsequently, the next focus of this study is to look at the impact of the OL-GDP model from the financial perspective on insurance and annuity products. Impact on pricing and reserving of the actuarial products are demonstrated in the following section. The model allows for simple quantitative measures of those parameters in connections to mortality forecasts. This is crucial in developing long-term mortality projections, which constitute an important issue in life insurance and pensions. A reliable and robust model provides an accurate forecast of mortality rates and an efficient pricing strategy that may address the issues of under-estimation of longevity risk and under-pricing of the financial products.

The rest of this chapter is organised as follows. Section 3.2 presents an analysis of the impact of various factors involved in the OL-GDP model. Based on the outcomes of Section 3.2, the sensitivity analyses are presented in Section 3.3. This section presents the impact of changes of the parameters of the OL-GDP model and other mortality models under study over insurance pricing for selected actuarial products. Section 3.4 presents the results of actuarial reserving for selected actuarial products. Finally, Section 3.5 concludes the whole discussion of this chapter.

3.2 Impact of Various Factors Involved in the OL-GDP Model

Socio-economic factors such GDP, inflation, unemployment have clearly been observed to have a causal effect on mortality experience. Major studies showed that improvements in mortality are accompanied by GDP growth. These co-movements have lasted for many years and are unlikely to be coincidental. Evidently, Boonen and Li (2017) claimed that the long run correlations between mortality developments and economic growth have been empirically studied before. In this section, the impact of various factors involved in the OL-GDP model is examined. Factors like age range, correlation coefficients, cg_x, age parameter and time dependent factor k_t^2 are being analysed.

3.2.1 Age Range

In the previous chapter, this study focused on the age range between 0–89 years old. To further study the impact and the sensitivity of the age range against the model, this study considers other age ranges such as 20–89 and 50–89 years old of male populations only. This study uses the common measures like MAPE, MAD and BIC to test the quality of fittings using the different sets of age ranges.

Surprisingly, consistent results were recorded for each measure of MAPE, MAD and BIC for each country. MAPE and BIC are fitting best for age range of 50–89 years old for all countries, while MAD measured well for age range of 0–89 years old across all countries.

As reflected by the OL-GDP model, older ages generate more significant results of the mortality rates. This is because, most of the time dependant factors become insignificant at an old age. Contrarily, the GDP effect will be more significant towards the time dependant factor of k_t^2 at an old age. Tables 3.1, 3.2 and 3.3 display the results of quality fittings over the different ranges of age for all nine Eurozone countries selected.

MAPE			
Country/Age	0-89	20-89	50-89
Austria	9.55%	5.53%	**3.02%**
Belgium	8.19%	4.75%	**2.36%**
France	5.19%	3.13%	**1.31%**
Germany	4.29%	2.81%	**1.15%**
Greece	9.35%	5.08%	**2.89%**
Italy	5.34%	3.47%	**1.24%**
Netherlands	7.20%	4.12%	**2.00%**
Portugal	8.26%	5.17%	**2.78%**
Spain	6.78%	4.39%	**1.99%**

Table 3.1: MAPE results for different age ranges of Male populations

MAD			
Country/Age	0-89	20-89	50-89
Austria	**0.001137**	0.001448	0.002133
Belgium	**0.000882**	0.001090	0.001592
France	**0.000681**	0.000794	0.001090
Germany	**0.000652**	0.000777	0.000914
Greece	**0.000702**	0.000889	0.001459
Italy	**0.000527**	0.000622	0.000946
Netherlands	**0.000597**	0.000748	0.001086
Portugal	**0.001288**	0.001488	0.002084
Spain	**0.000769**	0.000921	0.001558

Table 3.2: MAD results for different age ranges of Male populations

BIC			
Country/Age	0-89	20-89	50-89
Austria	-19695	-16013	**-10088**
Belgium	-19797	-16302	**-10336**
France	-28794	-21759	**-11977**
Germany	-22217	-17668	**-9921**
Greece	-10961	-9163	**-5856**
Italy	-27443	-21878	**-12110**
Netherlands	-19863	-16170	**-10300**
Portugal	-22768	-16937	**-10302**
Spain	-27243	-21345	**-11978**

Table 3.3: BIC results for different age ranges of Male populations

3.2.2 Correlation Coefficients

As the model incorporates the effect of GDP on mortality, this study denotes this factor as cg_x, a correlation coefficient between GDP and mortality rates. In the previous chapter, this study investigated the correlations between the average mortality rates (at individual ages) with the GDP fluctuations. The results showed that all countries across all panels of male, female and unisex generated highly negative correlations. This is in line with the findings from previous studies on the negative relationship between mortality experience and GDP performance. Moreover, as almost all of the findings are well-compared with the previous studies, it is important to note that correlations for older ages are extremely strong and stable for all genders.

In this chapter, this study investigates the impact of correlations between mortality experience and GDP performance for different time periods of Eurozone countries, Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain for male populations only. Results in Figure 3.1 present various time periods of 23-year, 33-year, 43-year and 53-year period used for all countries under study, except for Germany (13, 23, 33 and 43 years) and Greece (12, 22 and 32 years) of time period respectively.

As reflected clearly by all nine countries, longer time periods produce stronger correlations as compared to the shorter periods. This is because, longer periods provide more data that makes the coefficients correlate more in the long run. In general, strongest correlations were observed for all countries for the time period 1960–2012, except Germany (1970–2012) and Greece (1981–2012). Moreover, highest correlations were recorded for young ages (0–15 years old) and medium ages (of above 40 years old). Countries like Austria, Belgium, France, Germany and the Netherlands show persistent correlations over the period of 1960–2012. Italy, Portugal and Spain recorded similar trends of correlations with weak results particularly at the range between 25 to 30 years old. Peculiar results for Greece might be due to the shorter period of data availability. However, there is a strong negative correlation at 60 years of age and over.

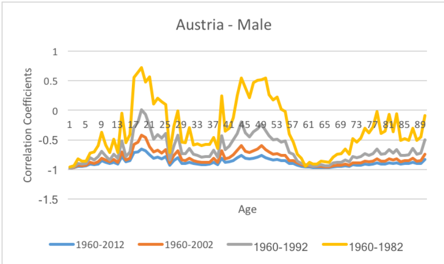
3.2.3 Age Parameter

As reflected in the OL-GDP model, the age parameter acts as a tool to limit the age range of the population under study. In the previous chapter, this study used the average age, $\bar{x}$ as the age parameter. In this subsection, this study extends the research to various numbers of age parameters of 20, 40 and 60 years old for Male populations only. This study uses the common measures of MAPE, MAD and BIC to test the quality fittings for various age parameters. In general, age parameter of 20 years old produce better results as compared to 40 and 60 years old.

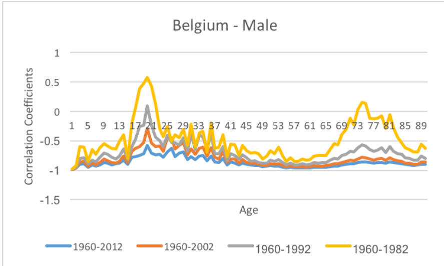
The impact of age parameter varies for each country. The results can be categorised into two groups, the young age, 20 and the old age 60. For instance, age parameter of 20 years old, suits Belgium, France, Greece, Italy, Portugal and Spain best. Meanwhile, Germany, Austria and Netherlands suit the 60 years old age parameter best. Tables 3.4–3.12 display the results of MAPE, MAD and BIC for various age parameters for all the nine countries under study. Based on these findings, the age parameter of 20 gives the best simulation for most countries as it provides better interpretation of the GDP effect by eliminating the effect from other time dependent factors of k_ts.

Austria				
Test/Age	20	40	60	Average
MAPE	9.55%	9.56%	**9.54%**	9.55%
MAD	0.001143	0.001138	**0.001133**	0.001137
BIC	**-19644**	-19685	-19731	-19696

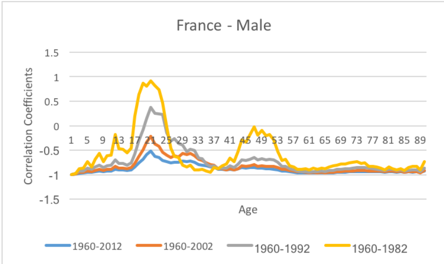
Table 3.4: Fitting Results of Austria by Age Parameters, Males



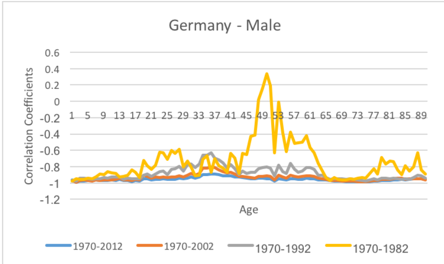
(A) Austria



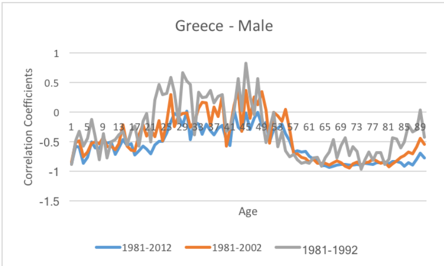
(B) Belgium



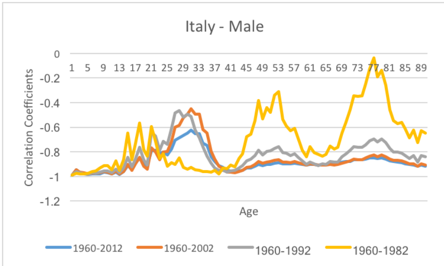
(C) France



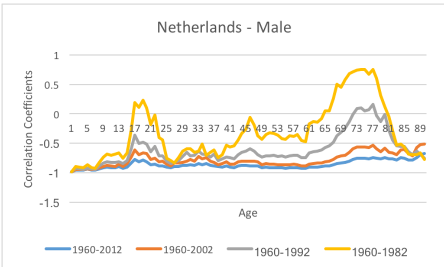
(D) Germany



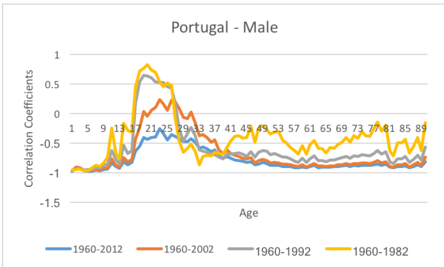
(E) Greece



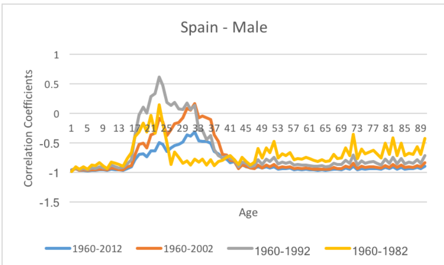
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

FIGURE 3.1: Correlation Coefficients between the Logarithm of GDP and Mortality Rates for Male Populations of nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain.

Belgium				
Test/Age	20	40	60	Average
MAPE	**8.16%**	8.18%	8.21%	8.19%
MAD	0.000884	0.000883	**0.000882**	0.000882
BIC	**-19772**	-19792	-19815	-19797

Table 3.5: Fitting Results of Belgium by Age Parameters, Males

France				
Test/Age	20	40	60	Average
MAPE	**5.13%**	5.18%	5.23%	5.19%
MAD	**0.000677**	0.000680	0.000687	0.000682
BIC	**-28502**	-28732	-29030	-28793

Table 3.6: Fitting Results of France by Age Parameters, Males

Germany				
Test/Age	20	40	60	Average
MAPE	4.29%	4.29%	**4.29%**	4.29%
MAD	0.000656	0.000653	**0.000650**	0.000650
BIC	-22255	-22224	**-22195**	-22225

Table 3.7: Fitting Results of Germany by Age Parameters, Males

Greece				
Test/Age	20	40	60	Average
MAPE	**9.23%**	9.31%	9.68%	9.35%
MAD	**0.000679**	0.000695	0.000724	0.000702
BIC	**-10949**	-10953	-11064	-10962

Table 3.8: Fitting Results of Greece by Age Parameters, Males

Italy				
Test/Age	20	40	60	Average
MAPE	**5.32%**	5.33%	5.36%	5.43%
MAD	**0.000515**	0.000525	0.000539	0.000563
BIC	**-27325**	-27415	-27555	-27852

Table 3.9: Fitting Results of Italy by Age Parameters, Males

Netherlands				
Test/Age	20	40	60	Average
MAPE	7.21%	**7.19%**	7.20%	7.19%
MAD	0.000635	0.000602	**0.000585**	0.000595
BIC	-19968	-19874	**-19848**	19861

Table 3.10: Fitting Results of Netherlands by Age Parameters, Males

Portugal				
Test/Age	20	40	60	Average
MAPE	8.27%	8.27%	**8.26%**	8.26%
MAD	**0.001276**	0.001285	0.001297	0.001288
BIC	**-22747**	-22764	-22784	-22766

Table 3.11: Fitting Results of Portugal by Age Parameters, Males

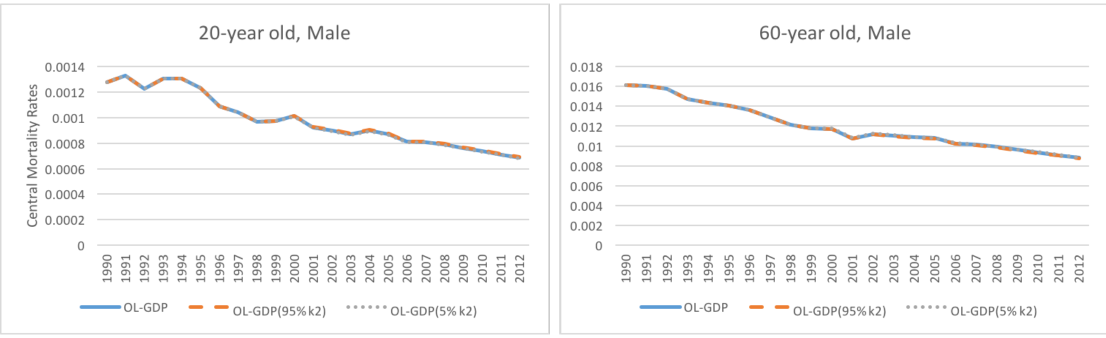
Spain				
Test/Age	20	40	60	Average
MAPE	**6.78%**	6.78%	6.79%	6.78%
MAD	0.000774	**0.006769**	0.000773	0.000769
BIC	**-27185**	-27226	-27318	-27243

Table 3.12: Fitting Results of Spain by Age Parameters, Males

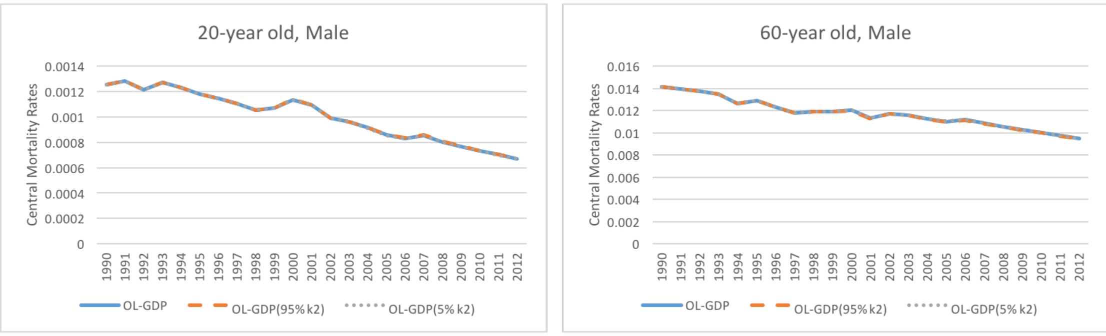
3.2.4 Estimation of k_t^2 Factor

As discussed earlier in Subsection 3.2.1, the model includes the additional factor of GDP. This additional factor acts as a vector of correlation coefficients between the logarithm of GDP per capita and mortality rates for ages between 0–89 years old. The impact of the GDP vector was tested against the age dimension and the time-dependent factor of k_t^2. The k_t^2 factor allows for variations of age that reflect the historical observations of rates improvement for different selection of ages.

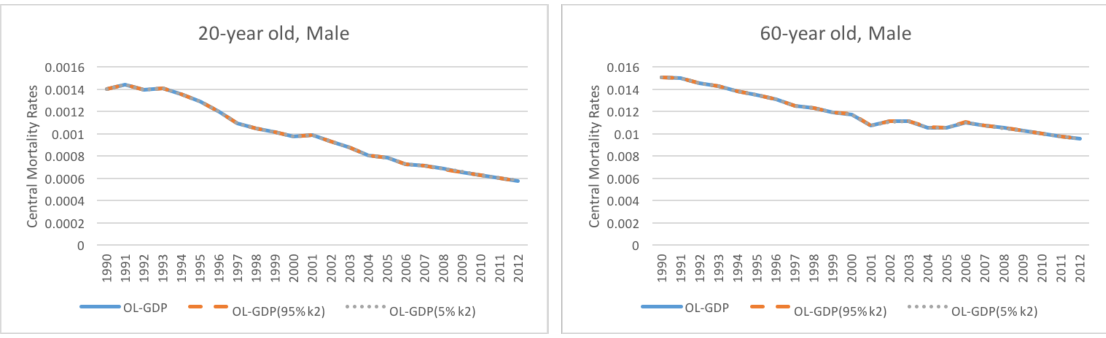
In this subsection, this study estimates the variations of k_t^2 factor at 5% intervals. The data used in this estimation is based on the period range of 1960–2012. The variations of the k_t^2 were tested on the 2000-2012 range of data for male populations age 20 and 60 years old respectively. Figure 3.2 displays the central mortality rates for male populations age 20 and 60 years old respectively for all the nine countries. This study observed that variations of k_t^2 do not have significant impact on the mortality rates especially for younger ages. For older ages, like 60 years of age, small changes of mortality rates were observed for some countries (Italy and the Netherlands). Other countries' mortality rates, remain unchanged. This shows that, the impact of GDP does not really affect any particular age of the populations, as it impacts the population as a whole for the specified time period under observation.



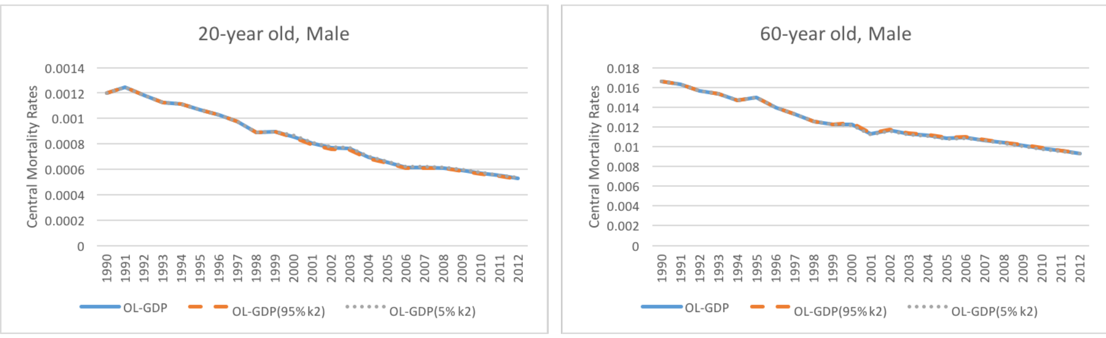
(A) Austria



(B) Belgium



(C) France



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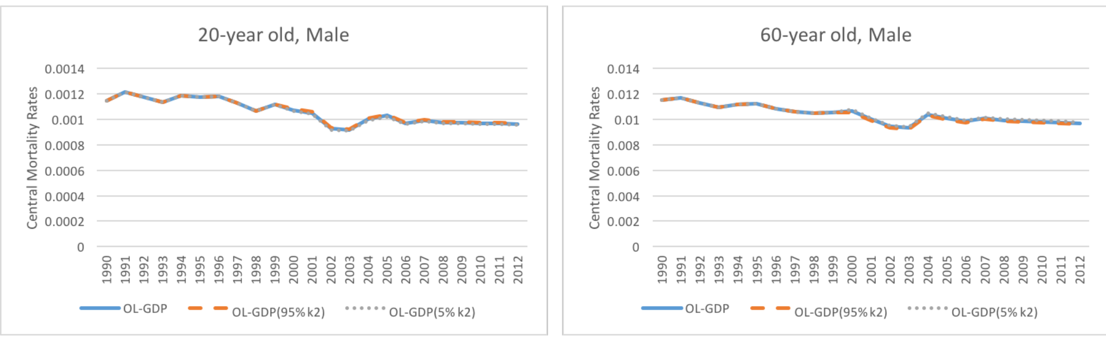

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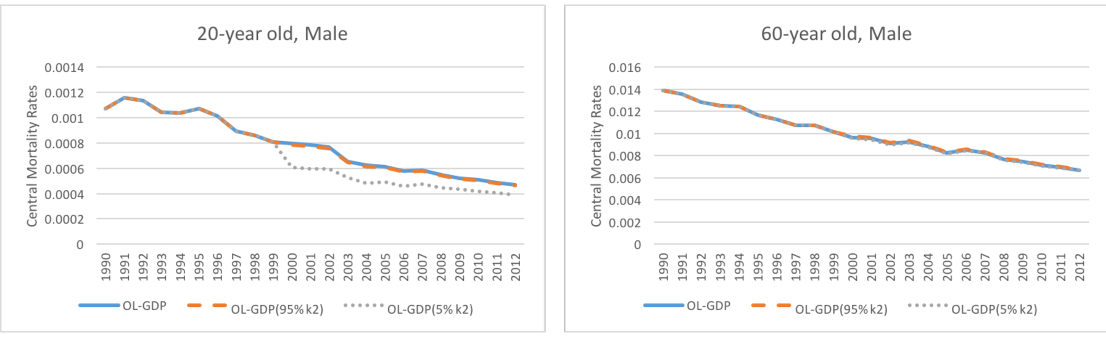

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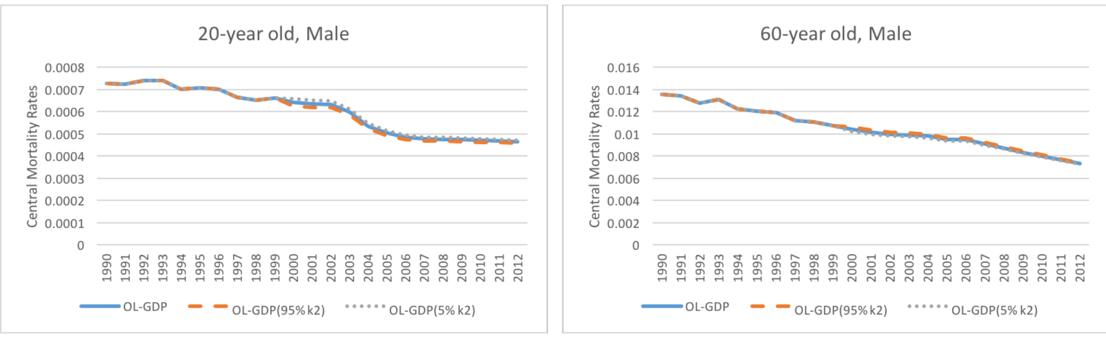

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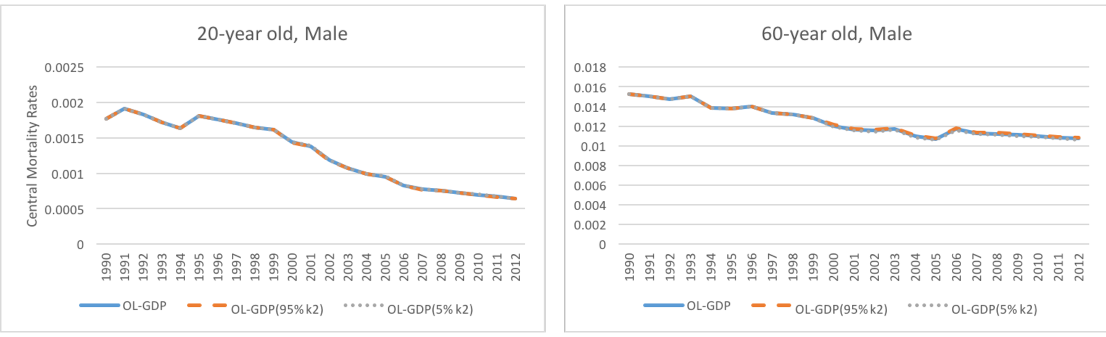

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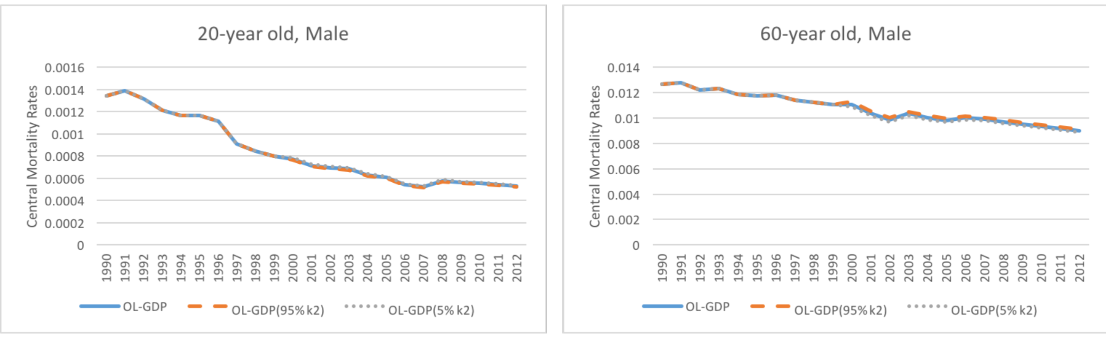


(I) Spain

FIGURE 3.2: Central Mortality Rates for Male Populations (1960–2012) for Different Values of k_t^2 from 2000–2012, for Ages 20 and 60 years old respectively.

3.2.5 Fitting and Forecasting of OL-GDP model

Besides the characteristics of factors used in the model formulation, the choice of forecasting methodologies also influence the projection of the mortality rates. Different forecasting tools tend to produce different projected rates. In this study, it considers the simple Naive[1], Arima[2], the Exponential Smoothing (ES)[3] and the Cubic Smoothing Spline (CSS)[4] as the methods used for forecasting. Naive method is used as a benchmark against other sophisticated forecasting methods like Arima, ES and CSS. ES approach is based on the weighted average of past observations, with the older observation weights declining exponentially; meaning, the most recent observations have higher weights (Hyndman and Booth, 2008). The Arima technique uses a variation between seasonal and non-seasonal data which combines unit root tests and returns the best model accroding to either Akaike Information Criterion (AIC) or Bayesian Information Criterion (BIC) values. The CSS approach, uses the spline interpolation method, in which the interpolant used is a special type of piecewise polynomial function. Again, in this subsection, the impact of the forecasting methods was tested on the estimated data derived from the OL-GDP model against the fitted data extracted from the HMD. The variations were tested on the male populations aged 40 years old for all the nine countries.

The fitting period is 48 years (1960–2007), (except Germany and Greece, for which it is 38 and 27 years respectively) and the forecasting period is 20 years (2008–2027). To be consistent with the previous exercise, this study uses the R package named “forecast” to estimate the future values of those factors involved in the model. This “forecast” package is fully automated in searching over the best model within the constraints provided.

[1]The Naive method is the simplest method of forecasting. The forecasting values are equivalent to the last observed value.

[2]Autoregressive Integrated Moving Average (ARIMA) used to predict future points in the series. The AR part involves the regression of its own lagged values. The MA part indicates regression errors that occurred at various times in the past. The I (Integrated) indicates the difference between their values and the previous ones. The combination of these features is to make the model fit the data as well as possible.

[3]Exponential smoothing (ES) is a technique for smoothing time series data. Particularly, the simple moving averages are weighted equally while the weights used are decreasing exponentially over time.

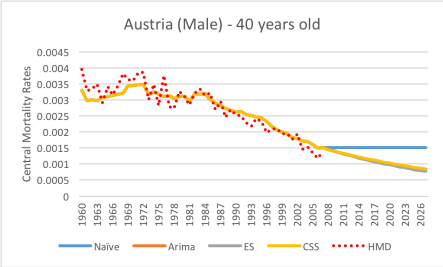
[4]The smoothing spline is a method of fitting a smooth curve using a spline function. Spline is better than polynomial as it produces less error during the interpolation process.

Figure 3.3 presents the forecasting results of central mortality rates for all countries. In general, all three methods, Arima, ES and CSS, generated better forecasts of mortality rates compared to Naive for all countries except Greece. Due to data limitations, Greece produced strange results for each method of forecasting. For Greece, ES technique generated quite similar results as Naive. While Arima and ES tend to forecast upward. Consistent forecasting results were generated by Arima, ES and CSS for Austria and Italy. The difference among these three methods are relatively small. On another note, in general, both Arima and ES methods produced similar forecasting results for all countries except for the Netherlands and Greece. Overall, it is interesting to note that all methods (except Naive) are producing downward forecasts of mortality trends for all countries except Greece.

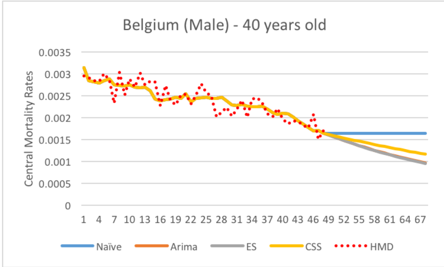
3.3 Actuarial Pricing Methodologies

Mortality models have been commonly used as bases for projecting mortality rates. An accurate projection of mortality rates is deemed critical in planning for organisation's sustainability. Some important classes of liabilities lie in organisations like life insurance and pensions as they are sensitive to the direction of future mortality trends. They rely their pricing and funding strategies diligently on prudent life tables. These life tables are based on assumptions that future death rates are known with certainty.

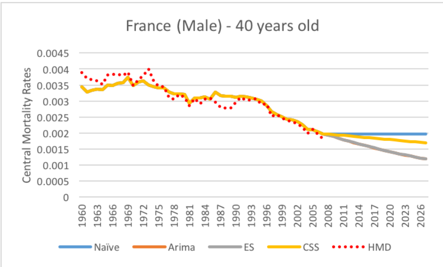
Developments in longevity improvements over the last few decades have led to uncertainty and ambiguity of future scenarios in planning and managing the mortality and longevity risks. Life insurers, pension funds and annuity providers are increasingly aware of their exposure to the risk of mortality changes and the need for better models for risk management (Sherris and Wills, 2008). The evolution of longevity risk has also modified the standard curve of life expectancies, underestimating the future ones (Barrieu et al., 2012). Issues like insufficient funds and inadequate protections are crucial in promoting a companys solvency. Life insurers and pension fund providers are increasingly mindful of the longevity risk and the needs for vigilant models of risk management. Actuarially, their mortality models need to be integrated into financial models in pricing their insurances and reserving their funds diligently.



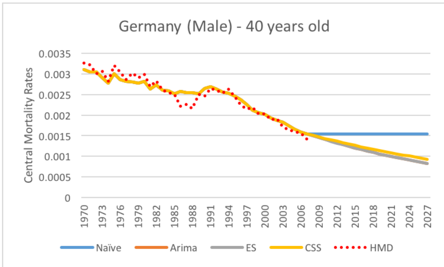
(A) Austria



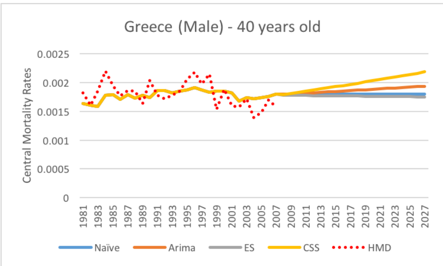
(B) Belgium



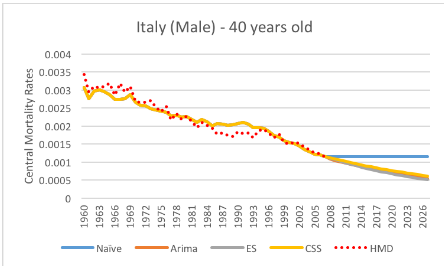
(C) France



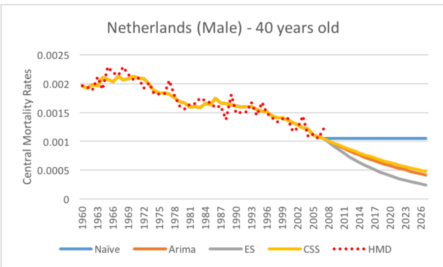
(D) Germany



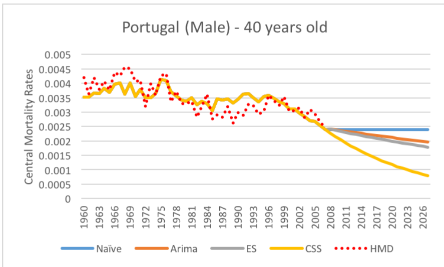
(E) Greece



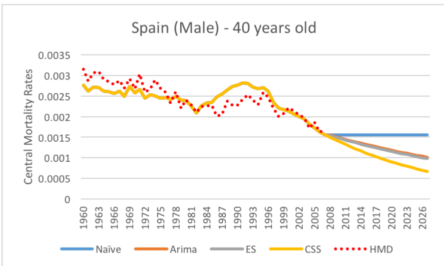
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

FIGURE 3.3: Fitted mortality rates of HMD data for Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain from 1960–2007 (except Germany: 1970–2007 and Greece: 1981–2007) and followed by the forecasted mortality rates from 2008–2027 for various methods of forecasting.

In the previous section, this study considered the impact of the GDP-age dependent mortality model, OL-GDP given the variations on its parameters. Following this, by reflecting the variations and range of modifiable factors involved in the model, this study believes that these may have an impact on the actuarial present value[5] (APV) of life annuities and life insurances. Factors like choices of data period and forecasting methods, also variations of correlation coefficients, interest rates, age groups and age parameters will be observed in this section.

Mainly, life insurance products offer two types of services; income replacement for premature death and long-term savings instruments. Different types of policies offer different coverage options and choices of investment to policyholders. Policies that offer the mortality coverage only are also known as term insurance. Meanwhile, those policies that combine the mortality coverage with a savings element are known as whole life and endowment insurances. Pure endowment is a type of insurance that offers a long-term savings only. Under this policy, the policyholder will get the benefits at the end of the policy period if the insured is alive. If the insured has died, there is nothing paid in the form of benefits. Policies that offer combinations between mortality coverage with savings element are typically earning interest, which is returned to the policyholder through policy dividends, cash values on termination of the policy or endowment sums on the policy maturity.

Besides the life insurance products, life insurers also sell annuity products. Annuities are contractual financial products that are designed to accept and grow funds from the annuitant. The period when an annuity is being funded and before pay outs begin is referred to as the accumulation phase. Upon annuitisation, annuitants will receive a lump sum or periodic payments until their death. Once the payments commence, the contract is in the annuitisation phase. At this point of time, insurers undertake risks associated with the longevity of the annuitant.

For the most part, this study considers four basic actuarial products namely term annuity, term insurance, endowment insurance and pure endowment insurance. The details of these products are described below:

(a) Term annuity due n-year annuity due where payments of £1 is made at the beginning of the year while an individual is alive for at most n years;

[5] Actuarial Present Value (APV) is the expected present value of a contingent cash flow stream, i.e., a series of payments that may or may not be made. APVs are typically calculated for the benefit-payment or series of payments associated with life insurance and life annuities.

$$\ddot{a}_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k {}_k p_x, \tag{3.1}$$

(b) Term insurance – n-year term life insurance where the death benefit payment of £1 is payable if the insured dies within the policy term;

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k {}_k p_x q_{x+k}, \tag{3.2}$$

(c) Pure endowment insurance – n-year pure endowment policy where the benefit payment of £1 is payable at the end of the policy term if the insured is alive with nothing payable in case of prior death (before maturity);

$$A_{x:\overset{1}{\overline{n}|}} = v^n {}_n p_x, \tag{3.3}$$

(d) Endowment insurance – n-year endowment policy where the benefit payment of £1 is payable to the beneficiaries if the insured dies during the policy term or to the insured on maturity of the policy if the insured survives the term, whichever occurs first (it is a combination of the n-year term life insurance and n-year term pure endowment insurance);

$$A_{x:\overline{n}|} = \sum_{k=0}^{n-1} v^k {}_k p_x q_{x+k} + v^n {}_n p_x, \tag{3.4}$$

This study assumes deaths are uniformly distributed in the year of age, where the relationship between the probability of death (as formulated in the APV formulae above) and the central mortality rates (as generated by OL-GDP mortality model) is as follows, see Bowers et al. (1997) for details:

$$q_x = \frac{m_x}{1 + \frac{1}{2} m_x}, \tag{3.5}$$

The annual effective interest rate,i, is assumed to be 3%, unless otherwise stated. The discount function,v, is given by the formula:

$$v = \frac{1}{1+i}, \tag{3.6}$$

In the following subsection, this study compares the APVs of the above mentioned financial products against the various factors reflected within the mortality models. This study bases its analysis on the Mean Percentage Error measure (MPE), which is defined as follows:

$$MPE = \frac{APV(estimated\ mortality\ rates) - APV(mortality\ rates)}{APV(mortality\ rates)}, \tag{3.7}$$

3.3.1 APV of Various Mortality Models

In this subsection, this study analyses the MPE results of four actuarial products for nine Eurozone countries. The analysis is based on the actuarial present values (APVs) for each actuarial product namely, annuity, term insurance, pure endowment and endowment. This study considers a term period, $n = 20$ years and the effective rate of interest, $i = 3\%$. This study further compares the MPE results of the new model, OL-GDP against three other mortality models, Lee and Carter (LC), O'Hare and Li (OL), Niu and Melenberg (LC-GDP). The analysis is conducted for male data only over the fitting period of 1960–2007 (except Germany and Greece, for which the fitting period is 1970–2007 and 1981–2007 respectively).

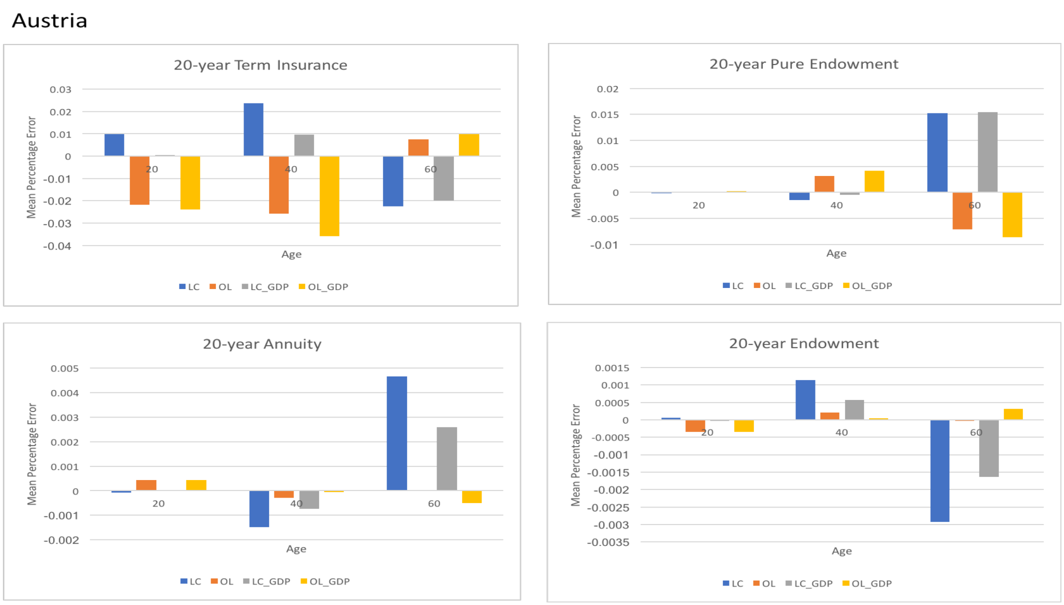


FIGURE 3.4: Austria – Actuarial Present Values (APVs)

Figure 3.4 presents the MPE results of the APV for a 20-year term insurance, annuity, pure endowment insurance and endowment insurance of Austria. The results display the MPEs of male population ages 20, 40 and 60 years old. Different actuarial products produces different results of MPE. In most cases, APVs for young ages of 20 years old give the smallest error for all products as compared to a 60-year old male population of Austria. The new GDP-age dependent model, OL-GDP calculates the APV better for Annuity and Endowment products, while for Pure Endowment product, OL model performs slightly better than the new model, OL-GDP. For Term product, most of the models perform moderately across all ages.

FIGURE 3.5: Belgium – Actuarial Present Values (APVs)

For Belgium (please refer to Figure 3.5), the MPE results of the four actuarial products can be categorised into two groups. For instance, annuity and pure endowment products produce similar trends of the APVs.The APVs are over-estimated at older ages of 60 years old, whilst for term and endowment products, the APVs are under-estimated for 60 years old age. Overall, our model, OL-GDP gives the smallest and most consistent range of errors of APVs for all four products.

FIGURE 3.6: France - Actuarial Present Values (APVs)

As presented in Figure 3.6, in general, the new model, OL-GDP provides the smallest and most consistent range of errors of APV calculations for all four products across all ages of 20, 40 and 60 years old for France. Better APV calculations are observed at the middle ages of 40 years old for all products. APVs are over- and under-estimated at age 60 for annuity, pure endowment and endowment products. For term insurance products, the over-estimations of APV are observed for young ages of 20 years old.

FIGURE 3.7: Germany - Actuarial Present Values (APVs)

For Germany, the calculations of APV display over- and under-estimations for medium to old ages of all the four products (Figure 3.7). Similar trends are observed for term and endowment products, and another similar trend for annuity and pure endowment products. For term and endowment products, all four mortality models, LC, OL, LC-GDP and OL-GDP under-estimate the APV calculations for ages 40 and 60 years old. Meanwhile, the APVs for annuity and pure endowment products are over-estimated for the same ages of 40 and 60 years old. Overall, our model, OL-GDP performs better as compared to other models.

Similar trends were observed for two groups of products, term and endowment,

FIGURE 3.8: Greece - Actuarial Present Values (APVs)

and annuity and pure endowment for Greece (please refer to Figure 3.8). For term and endowment, all APVs are under-estimated for all ages of 20, 40 and 60 years old. Significant range of errors is observed for term product. Whilst for annuity and pure endowment products, range of errors is more visible for 60 years old. At the same time, very small errors are detected for young age of 20 years old.

Figure 3.9 presents the APVs for Italy. Replicating Greece's results, similar trends are observed for the same groups of products, term and endowment, and annuity and pure endowment. In addition, under-estimations of APV are observed for term and endowment products and over-estimations for annuity and pure endowment products. The new model, OL-GDP seems to provide consistent calculations of APV for all products across all ages.

FIGURE 3.9: Italy - Actuarial Present Values (APVs)

FIGURE 3.10: Netherlands - Actuarial Present Values (APVs)

Results for the Netherlands are consistent for ages 20 and 40 years old for annuity, pure endowment and endowment products (please refer to Figure 3.10). Opposite trends are observed for age 60 years old of annuity and endowment products. Under-estimations occur at the ages of 20 and 40 years old for all mortality models of term and endowment products except for 40 years old of term insurance.

For Portugal (please refer to Figure 3.11), similar observations were noted like for the majority of the countries discussed. Opposite characteristics were reported for

Portugal

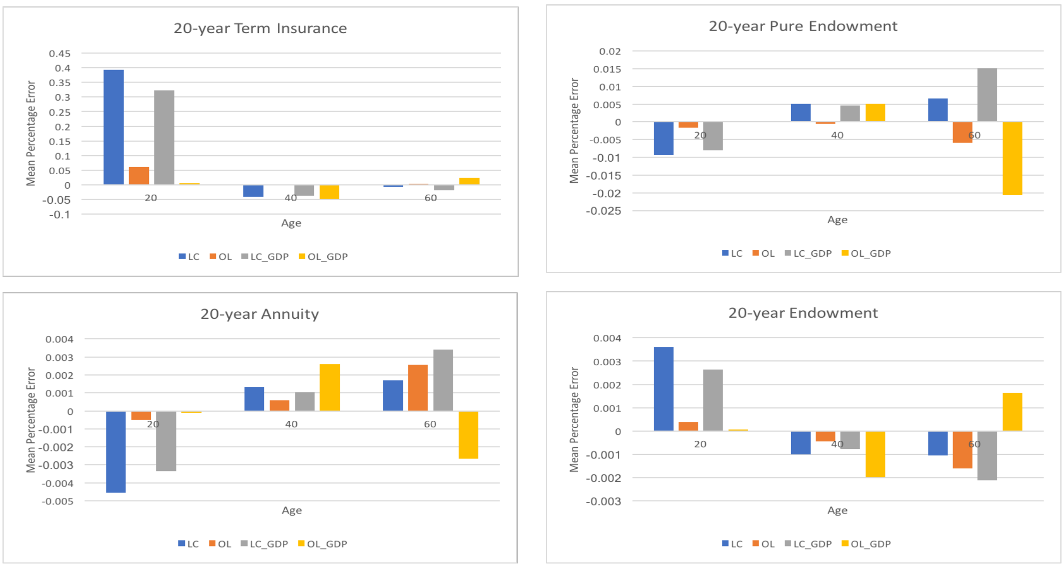


FIGURE 3.11: Portugal - Actuarial Present Values (APVs)

two groups of products, term and endowment, and annuity and pure endowment. Significant range of errors are spotted on the 20 years old male population across the products.

Spain

FIGURE 3.12: Spain - Actuarial Present Values (APVs)

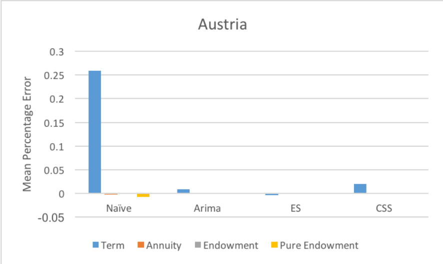
Results for Spain (please refer to Figure 3.12), are replicating closely the results of Portugal. Significant range of errors are observed at the age of 20 years old. Except for term product, there are also significant ranges of errors for 60 years old of annuity, pure endowment and endowment products. As before, the new model, OL-GDP tends to produce consistent measures of APVs for all products across all ages. The errors produced by OL-GDP are relatively small.

3.3.2 APV of Various Forecasting Methods

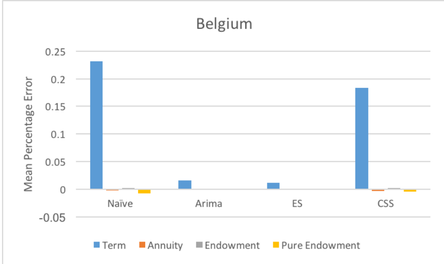
This subsection investigates the impact of the forecasting methods on the APVs of four actuarial products namely Annuity, Term Insurance, Endowment Insurance and Pure Endowment. As discussed in subsection 3.2.5, the choice of forecasting methodologies also influences the projection of the mortality rates. Different forecasting tools tend to produce different projected rates. Under this observation, the forecasting was done on the forecasted mortality rates in 2014. The forecasting methods used are Naive, Arima, ES and CSS. The interest rate, i used is 3%. This insurance and annuity products are issued to a male age of 30 years old, for the term period of 20 years across all products.

As presented in Figure 3.13, term insurance product provides the biggest errors for all forecasting methods across all countries. In addition, as reflected by all forecasting methods, the term insurance product is over-estimated in most of the countries except Germany, Italy, the Netherlands and Portugal. Among all methods, Naive produced the highest overestimated errors for all countries with the highest over-estimation being recorded for Spain at 0.5%. ES performs exceptionally well in Austria, where the MPEs for all products are almost zero. Other forecasting methods also performed well for Austria except for the Naive approach of term insurance product. For Belgium, both methods of Arima and CS are performing incredibly, where all products recorded almost zero results of the MPEs. Except for the the Netherlands and Greece, Arima and ES methods generate quite similar results of MPE for all products. CSS method produces the least error against other forecasting methods across all products in Germany, Italy and Spain. Under-estimations are significantly observed in Italy and Netherlands for all methods except Naive. In Portugal, the underestimation is recorded under CSS method for term insurance product.

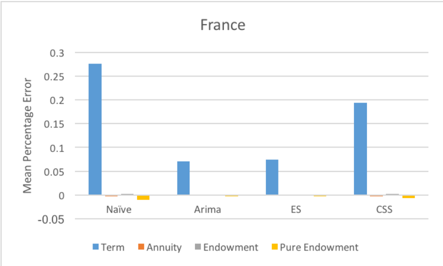
In general, with the exclusion of term insurance product, all of the forecasting methods perform exceptionally well for annuity, endowment insurance and pure endowment insurance across all countries.



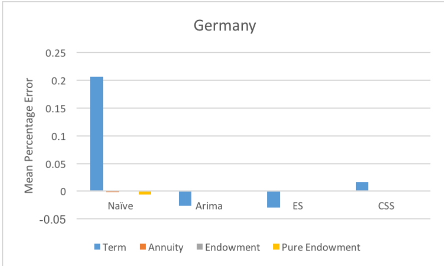
(A) Austria



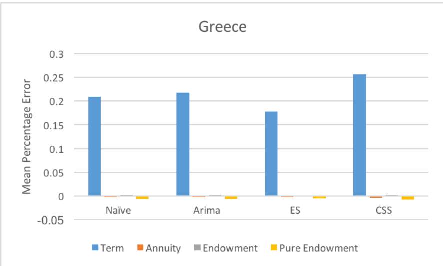
(B) Belgium



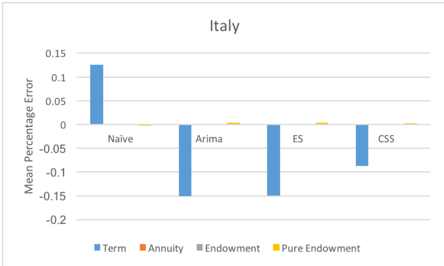
(C) France



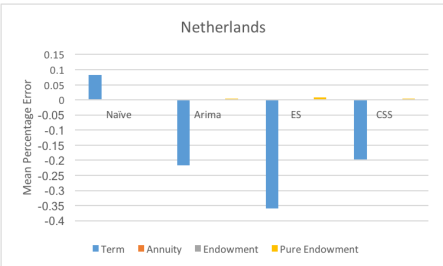
(D) Germany



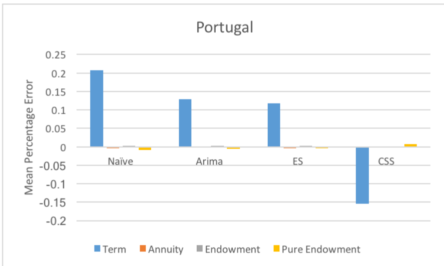
(E) Greece



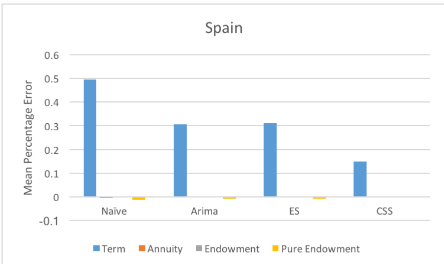
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

FIGURE 3.13: Mean Percentage Error of Actuarial Present Values (APVs) for a 30-year term of Annuity, Term Insurance, Endowment Insurance and Pure Endowment Insurance based on the OL-GDP model issued to Male Populations age 30 years old using Naive, Arima, ES and CSS Forecasting methods forecasted in 2014 against the actual value of HMD in 2014.

3.3.3 APV of Various Interest Rates

Small changes of interest rates result to significant impact over APV. Evidently, there is an inverse relationship between the interest rates and the present values. The gradients of the trends are depending on the age of the insured and also the length of the policy period. To verify this statement, this study examines the impact of the interest rates and policy tenure on endowment policy for a person ages 20, 40 and 60 years old respectively.

As recorded in Figures 3.14–3.22, higher interest rates tend to lower the values of the APV. The length of the policy also affects the values of the APV. In general, there are insignificant changes of interest rates for 20 and 40 years old. However, there is a significant gap on 60 years old trends.

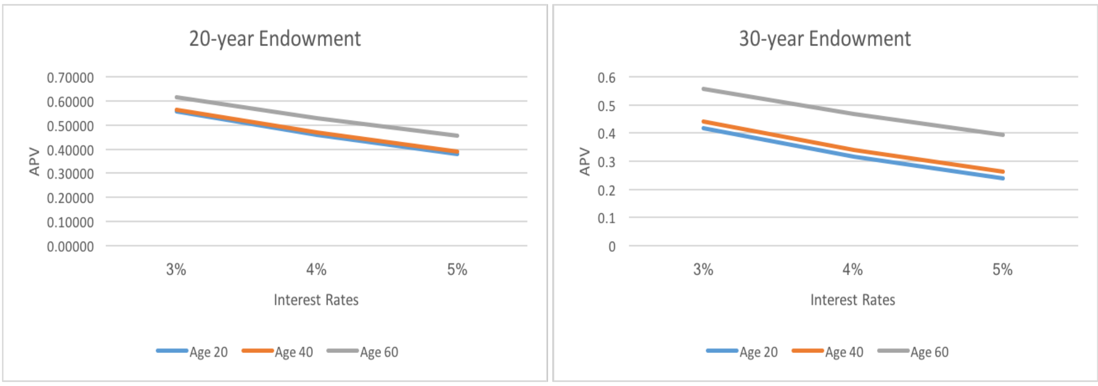


FIGURE 3.14: Austria - APV of Various Interest Rates, 3-5%

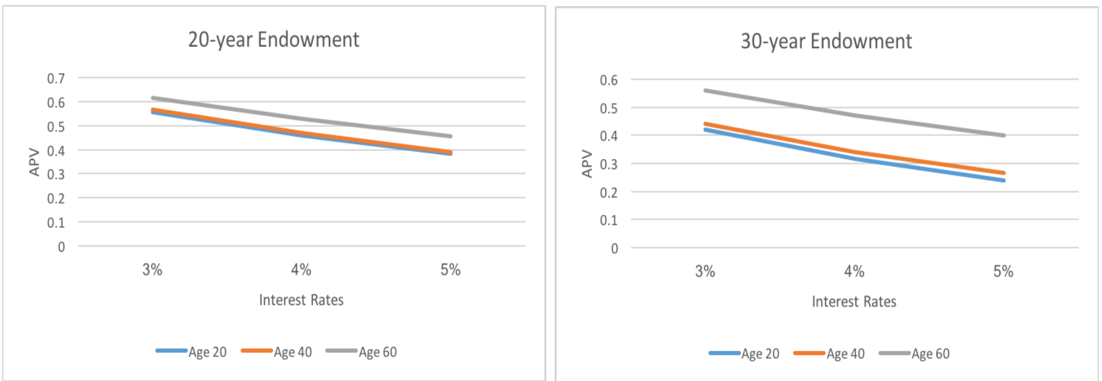


FIGURE 3.15: Belgium - APV of Various Interest Rates, 3-5%

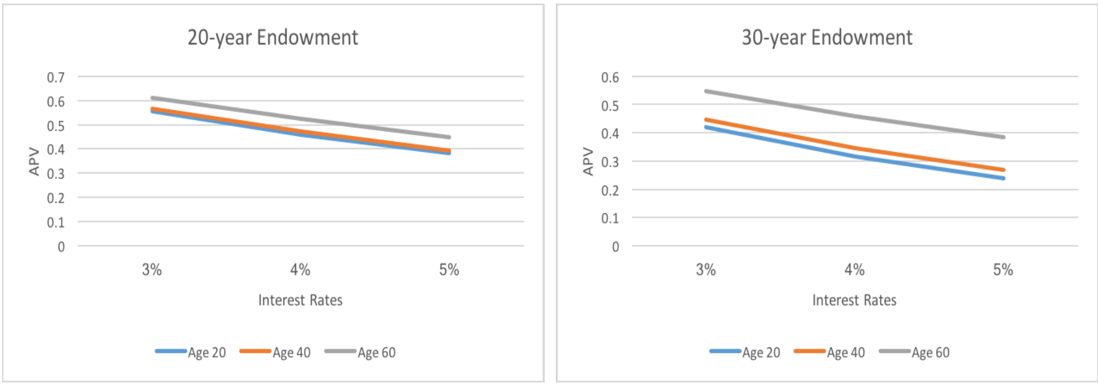


FIGURE 3.16: France - APV of Various Interest Rates, 3-5%

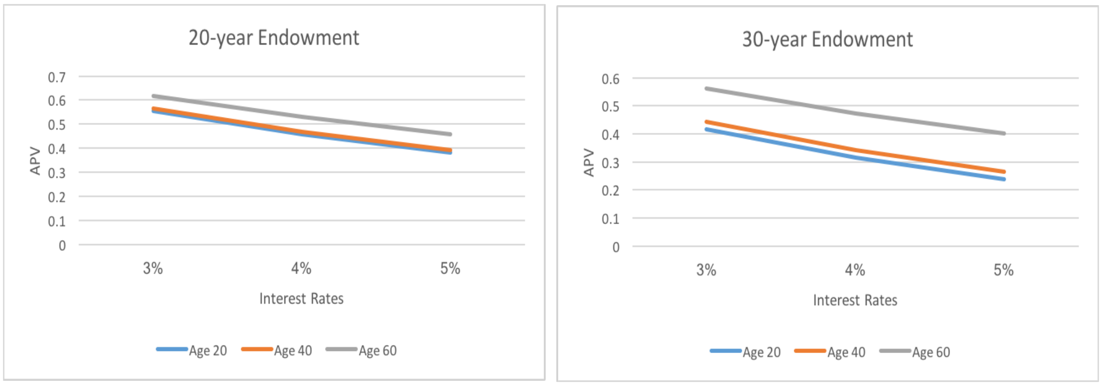


FIGURE 3.17: Germany - APV of Various Interest Rates, 3-5%

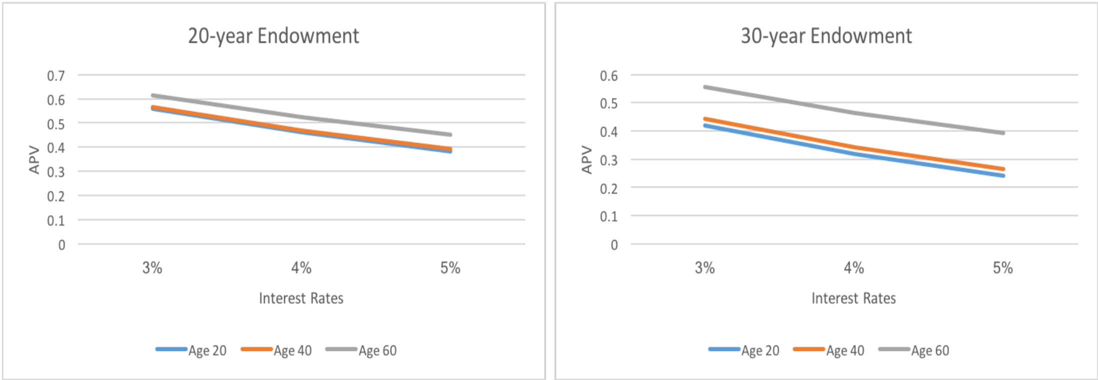


FIGURE 3.18: Greece - APV of Various Interest Rates, 3-5%

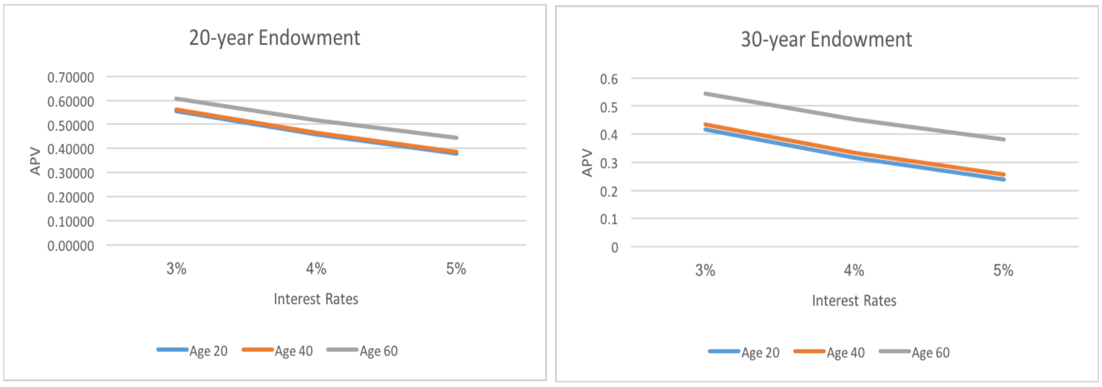


FIGURE 3.19: Italy - APV of Various Interest Rates, 3-5%

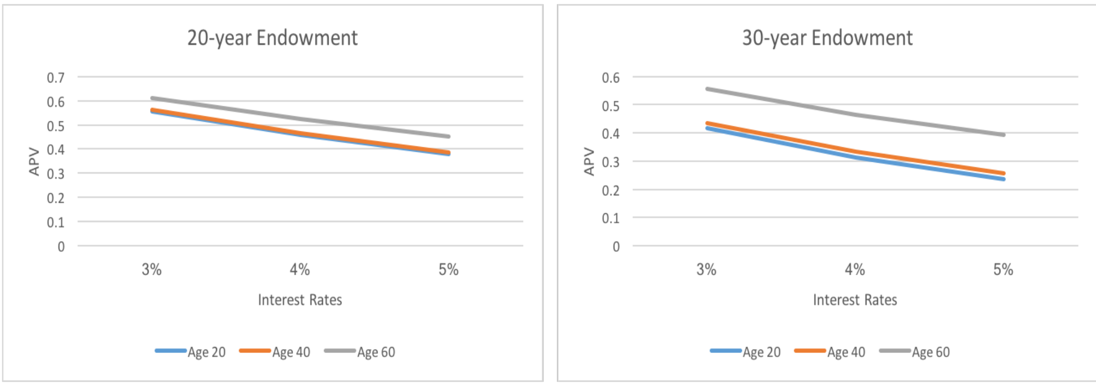


FIGURE 3.20: Netherlands - APV of Various Interest Rates, 3-5%

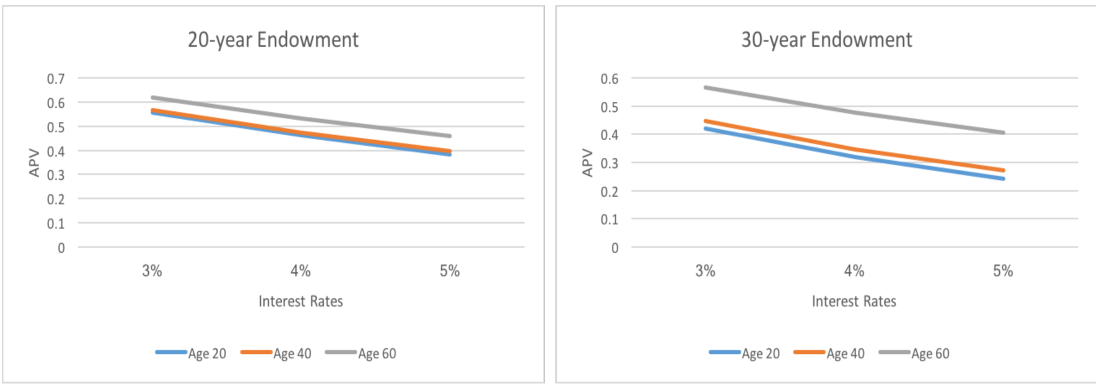


FIGURE 3.21: Portugal - APV of Various Interest Rates, 3-5%

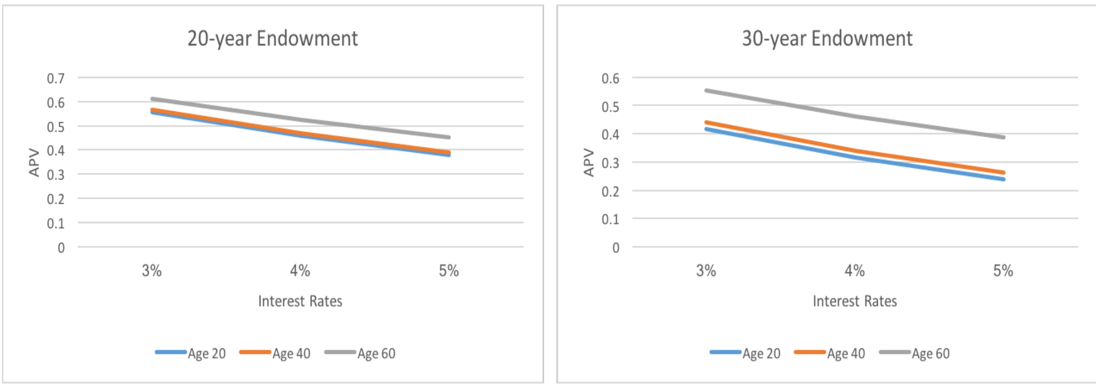


FIGURE 3.22: Spain - APV of Various Interest Rates, 3-5%

3.3.4 Annual Premium of Various Models

Appropriate choice of mortality tables is crucial in calculating the insurance premiums accurately. In this subsection, this study tabulates the annual insurance premiums of common life insurance products like Term, Endowment and Pure Endowment for the insurance term of 20 years. This study examines the annual insurance premium for entry ages of 30 and 60 years old for all the nine countries of the Eurozone. This study compares the annual insurance premiums against the four mortality models of LC, OL, LC-GDP and OL-GDP for better understanding the impact of mortality rates on the annual insurance premiums. Generally, the premiums are tailored to the respective country. Each country produces different results of the insurance premiums from different models of mortality.

Tables 3.13–3.21 present the illustrations of the annual premiums for 20-year life insurance products with £1,000 benefit payment, payable at the end of the year of death/maturity to a person aged 30 and 60 years old respectively. Focusing on the OL-GDP model only, this model priced the annual premiums of 20–year term insurance issued to a person ages 30 are between £1.05–£2.26 across the nine Eurozone countries. The lowest annual premium is recorded for Netherlands whilst the highest annual premium is recorded for Portugal. Similar results were recorded for a 20–year endowment issued to the individual aged 30. The annual premiums range between £36.59–£37.17, with the Netherlands and Portugal entailing the lowest and highest annual premiums respectively. Inverse results were recorded for a 20–year pure endowment, where the lowest annual premium was priced for Portugal (£34.90), whilst the highest annual premium was priced for the Netherlands (£35.53).

For insurances issued to an individual aged 60, the 20–year term insurance is priced between £20.10–£24.41. Italy had at the lowest annual premium and Portugal recorded the highest annual premium. For the 20–year endowment insurance, the range of the annual premiums recorded by OL-GDP is between £44.94–£47.16. Again, Italy revealed at the lowest annual premium and Portugal was priced at the highest annual premium. For the 20–year pure endowment, the lowest annual premium was recorded by Portugal with £22.75 and the highest annual premium was recorded by France with £24.98.

Austria, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.52	1.51	1.49	1.50
	Endowment	36.77	36.78	36.76	36.78
	Pure Endowment	35.24	35.27	35.27	35.27
60	Term Insurance	21.74	22.51	21.84	22.58
	Endowment	46.03	46.38	46.18	46.42
	Pure Endowment	24.29	23.86	24.34	23.84

TABLE 3.13: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Austria

Belgium, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.69	1.63	1.73	1.64
	Endowment	36.89	36.84	36.90	36.87
	Pure Endowment	35.19	35.21	35.16	35.22
60	Term Insurance	22.71	22.97	22.42	23.03
	Endowment	46.20	46.69	46.12	46.72
	Pure Endowment	23.48	23.72	23.70	23.69

TABLE 3.14: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Belgium

France, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	2.12	1.98	2.10	1.94
	Endowment	37.07	36.98	37.07	36.96
	Pure Endowment	34.95	35.00	34.97	35.02
60	Term Insurance	20.14	20.59	20.17	20.74
	Endowment	45.53	45.63	45.57	45.72
	Pure Endowment	25.38	25.03	25.40	24.98

Table 3.15: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for France

Germany, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.62	1.54	1.60	1.54
	Endowment	36.80	36.78	36.79	36.76
	Pure Endowment	35.17	35.23	35.18	35.22
60	Term Insurance	23.08	23.76	23.10	23.71
	Endowment	46.76	46.98	46.77	46.96
	Pure Endowment	23.68	23.22	23.66	23.25

Table 3.16: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Germany

Greece, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.77	1.79	1.77	1.74
	Endowment	36.93	36.94	36.93	36.93
	Pure Endowment	35.15	35.14	35.15	35.19
60	Term Insurance	21.82	21.97	21.82	21.92
	Endowment	45.85	46.17	45.85	46.15
	Pure Endowment	24.03	24.20	24.03	24.22

Table 3.17: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Greece

Italy, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.21	1.19	1.31	1.16
	Endowment	36.71	36.65	36.77	36.67
	Pure Endowment	35.49	35.46	35.46	35.51
60	Term Insurance	20.26	19.83	20.07	20.10
	Endowment	45.08	44.77	45.01	44.94
	Pure Endowment	24.82	24.93	24.94	24.84

Table 3.18: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Italy

Netherlands, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.06	1.00	1.09	1.05
	Endowment	36.59	36.54	36.61	36.59
	Pure Endowment	35.53	35.53	35.51	35.53
60	Term Insurance	22.68	22.38	22.20	22.21
	Endowment	45.83	45.96	45.62	45.86
	Pure Endowment	23.14	23.57	23.42	23.65

Table 3.19: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Netherlands

Portugal, £					
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	2.56	2.41	2.59	2.26
	Endowment	37.37	37.24	37.39	37.17
	Pure Endowment	34.81	34.82	34.79	34.90
60	Term Insurance	23.54	23.79	23.26	24.41
	Endowment	46.83	46.77	46.70	47.16
	Pure Endowment	23.29	22.97	23.44	22.75

Table 3.20: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Portugal

		Spain, £			
Age	Insurance Type	LC	OL	LC-GDP	OL-GDP
30	Term Insurance	1.84	1.63	1.84	1.55
	Endowment	36.99	36.86	36.99	36.80
	Pure Endowment	35.15	35.22	35.15	35.24
60	Term Insurance	20.94	20.96	20.85	21.40
	Endowment	45.62	45.53	45.59	45.84
	Pure Endowment	24.68	24.57	24.73	24.44

Table 3.21: £1,000 Benefit Payment is Payable at the End of Year of Death / Maturity to a Person Aged 30 and 60 years old, where n=20 years for Spain

3.4 Actuarial Reserving

In the insurance context, actuarial reserve is the present value of the future cash flows of an insurance policy and the total liability of the insurer is the sum of the actuarial reserves for each individual policy. In better terms, actuarial reserve is a liability equal to the actuarial present value (APV) of the future cash flows of a contingent event.

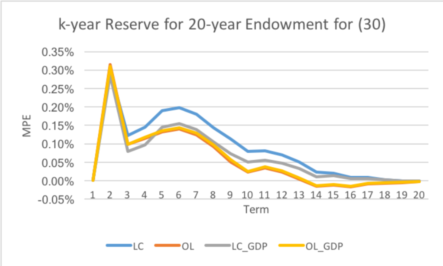
Sufficient reserves are crucial for all regulated insurers as they are required to keep aside reserves for managing future liabilities. Reserves play an important role in assessing the financial condition of an insurer. Reserves are also important in assessing the solvency of an insurer, in terms of its ability to meet its liabilities. Moreover, reserves are also important in pricing the insurance products more accurately. Actuaries price the insurance products by estimating the future cost of claims on risks yet to be paid off to the insured by extrapolating the past paid and reserved claim cost.

In this section, this study presents some results related to the calculation of the actuarial reserve. Generally, the benefit reserve is the difference between the APV of the insurance and the APV of future benefit premium at an annual rate of the premiums. The general formula of the actuarial reserve is:

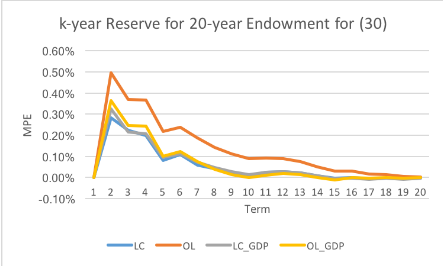
$$ {}_kV_x = A_{x+k} - P_x \ddot{a}_{x+k} \tag{3.8} $$

The illustration of calculating the reserve is based on a 20–year endowment policy for a person aged 30 years old, based on the Human Mortality Database of male populations for the selected Eurozone countries. The fitted mortality rates used during the calculation process are based on 2007 data. Figure 3.23 presents the mean percentage error of a k-year reserve of a 20-year endowment for a person aged 30 years old. This study compares the MPE results among the four mortality models, LC, OL, LC-GDP and OL-GDP.

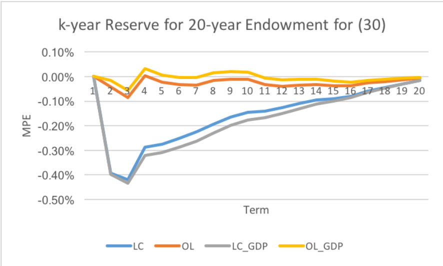
Interestingly, this study observed different trends of reserve accumulations presented by each country. However, countries like Austria and Belgium pose quite similar trends of reserves. Reserves for both countries are over-estimated by all the four models, with the least error being presented by the new OL-GDP model. Meanwhile, countries like France, Italy, Portugal and Spain behave similarly, with notable under-estimation of reserves being observed from LC and LC-GDP models conjointly. Anew, the model stands to provide the least error in estimating the reserves. Greece and the Netherlands are being over-estimated at the beginning of the policy inception and remain consistent in the middle of the tenure. MPE results for Germany are consistent for all four models. OL-GDP presents persistent results starting at the 8th year of the term. Generally, it is notable that OL-GDP presents the least error for all countries as compared to other mortality models.



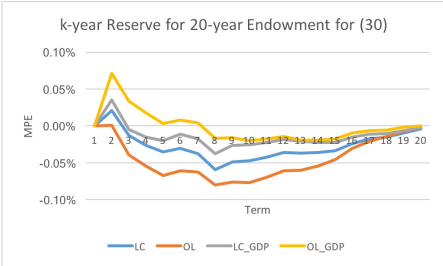
(A) Austria



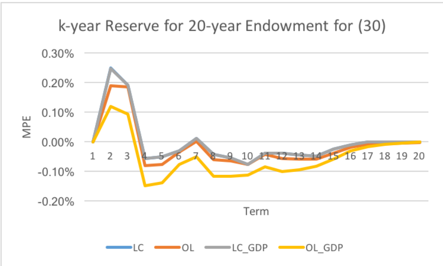
(B) Belgium



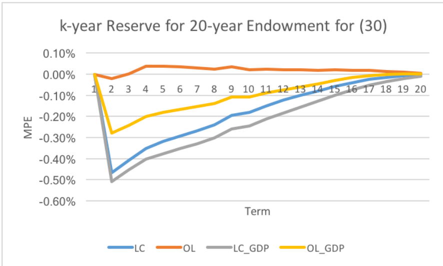
(C) France



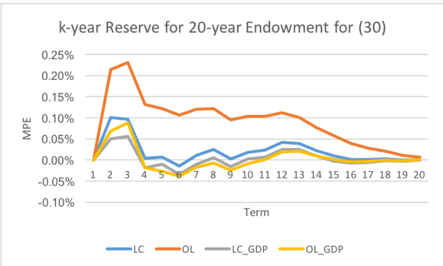
(D) Germany



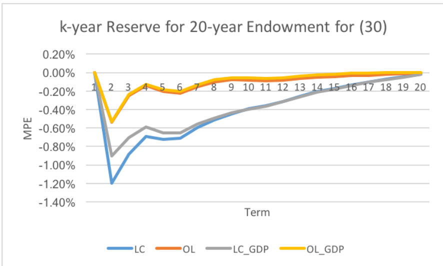
(E) Greece



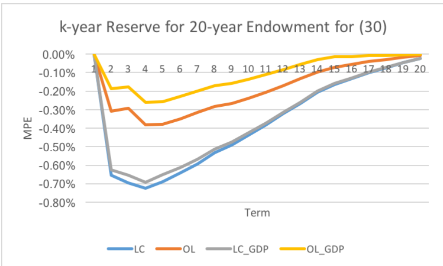
(F) Italy



(G) Netherlands



(H) Portugal



(I) Spain

FIGURE 3.23: Mean Percentage Error for a k-year Reserve for 20–year Endowment Policy for a Person Aged 30, based on Male Populations for nine countries: Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain.

3.5 Conclusion

In Section 2.4, this study developed a new model, OL-GDP, incorporating the GDP factor into the existing model of O'Hare and Li (OL). In this chapter, this study expands the investigation on the OL-GDP model, by examining its behaviour towards variations of parameters involved in formulating the model. This study conducts sensitivity analyses as part of assessing the impact of the model onto the parameters variations. The analysis includes the variations of the age range, varieties of correlation coefficients, different age parameters and also the variations of the time-dependent factor of k_t^2.

In addition, this chapter further extends the study by looking at the impact of the new model, OL-GDP from the financial perspective. Organisations like life insurers and pension fund providers are sensitive towards the change of mortality rates. Reflecting the variations and range of modifiable factors in the model, this study observed that these have impacted the actuarial present values (APVs) of the actuarial products. This study focuses its research on the basic products of life insurance and annuity, namely annuity due, term insurance, pure endowment and endowment insurance. Moreover, this study extends the investigation on actuarial reserves as well.

For better understanding in demonstrating the good aspects of the new model, OL-GDP, this study applied similar approach onto other mortality models of Lee and Carter (LC), O'Hare and Li (OL) and Niu and Melenberg (LC-GDP) comparatively. As mortality experience is unique across countries, different mortality models suit each country differently. However, in general, the new model, OL-GDP suits best most of the countries and scenarios. As the impact of mortality improvements is more significant to the insurers and pension fund providers, they need to be more mindful and vigilant in choosing reliable mortality models. Future investigation will certainly shed some light in mitigating the longevity risk.

Chapter 4

Hedging of Life Insurance and Annuity Mortality Risks

Fundamentally, life insurance policies and annuity contracts have opposite financial goals. Life insurance provides financial aids to beneficiaries due to unexpected or untimely death of the policyholder, while annuities provide a guaranteed stream of income for life specially catered for old ages. In parallel, the liabilities value of life insurance and annuity are also moving in opposite directions. The values of the liabilities are more sensitive in response to a change in the underlying mortality experience. In mitigating mortality risks, insurance and pension providers may hedge the risk naturally as this type of hedging does not require any sophisticated financial products like forwards or derivatives. The use of natural hedging is seen to reduce the financial risks of those instruments. Through natural hedging, the performance of both instruments will cancel each other out in stabilising the aggregate liabilities of the cash flows.

This chapter provides empirical evidence that improvement of mortality rates will impact the liabilities of life insurance and annuity portfolios. Technically, a decrease in mortality rates will have a negative impact on annuity liabilities whilst, an increase in mortality rate will impact for the life insurance portfolio. For this reason, insurers who are able to utilise natural hedging will have a competitive advantage. This approach is internal to the insurance company, which makes it more convenient and practical to implement by optimising the allocation of their annuities and life insurance to hedge against longevity risk. Literally, this method works well with a firm that writes both businesses, insurance and annuity. For insurers that write a single line of business, either insurance or annuity, its only portfolio of mortality risks is unlikely to provide an optimal mortality hedge.

4.1 Introduction

The moment when the aggregate change of mortality differs from that anticipated, it provides a risk factor to both life insurance and annuities by affecting their fair values, premiums and reserves. Relatively, the improvement of mortality from expectations, will decrease a life insurer's liabilities as the death benefit payments will be payable later than expected. Contrarily, this will cause the annuity provider to lose its relative advantage as they have to pay the annuity benefits longer than expected. However, if the mortality deteriorates, the situation is otherwise. Life insurers will face losses and annuity providers will have gains. In view of this, natural hedging makes use of the interaction between life insurance and annuity to balance out the liability outflows. The same changes of mortality experience provides opposite impacts between both instruments, life insurance and annuity (Cox and Lin, 2007).

The purpose of this chapter is to analyse the implementation of natural hedging on mortality risks and to propose mortality securitisation as a tool to manage the risks. Studies of natural hedging have been conducted earlier by many researchers like (Cox and Lin, 2007; Wills and Sherris, 2010; Wang et al., 2010, 2013). Some researchers studied the impact of mortality changes on life insurance (Marceau and Gaillardetz, 1999) and annuities (Frees et al., 1996; Milevsky and Promislow, 2001) separately, or investigated the combination of bonds and other mortality derivatives (Cairns et al., 2006a).

Marceau and Gaillardetz (1999) explored the reserves calculation in mortality and interest rates environment of a life insurance portfolio for term life insurance and pure endowment policies. On similar objective, Milevsky and Promislow (2001) calculated the mortality-contingent claims, by modelling the future interest rate risk. They observed that both mortality and interest rate risk can be hedged.

Subsequently, Wang et al. (2003) investigate the influence of changes in mortality factors and propose an immunisation model to hedge against mortality risks. Wang et al. (2013) investigate a natural hedging strategy and attempt to find an optimal allocation of insurance products using experienced mortality rates rather than population mortality data. They consider both variance and mispricing effects of longevity risk at the same time. According to Cox and Lin (2007), the insurance price is negatively related to the degree of natural hedging and natural hedging is an important factor contributing to the price difference among life insurers.

Heuristically, this chapter proposes mortality securitisation between capital market investors and annuity issuers. In principle, it transfers the risk out of the balance sheet or liabilities of the annuity issuer. This study investigates the impact of mortality securitisation on annuity portfolio and, consequently, addresses the issuers asset-liability management problem. If the annuity issuer manages to hedge its mortality risk successfully, the product's risk premium will be reduced, subsequently lowering its prices. Hence, this will improve its competitiveness in the market.

The rest of the chapter is organised as follows. Section 4.2, demonstrates the impact of the risks related to mortality changes on life and annuity portfolio. Natural hedging approach has been further illustrated to signify the idea of natural hedging and the needs for mortality securitisation, Section 4.3 analyse the economics of mortality risk and execute the mortality securitisation. Section 4.4 proposes the mortality securitisation framework for annuity portfolio and Section 4.5 concludes.

4.2 Risk of Mortality Change in Life and Annuity Portfolio

This subchapter illustrates the concept of natural hedging. Natural hedging has been widely undertaken by many companies. Companies are becoming increasingly skillful at combing economic analysis of their business with financial management. By exploiting the relationship between their earnings with their liabilities, they can reduce their exposure to economic or financial risks. Natural hedging is not only suitable for insurers and annuity providers, but it can be extensively extended to any line of businesses. Even, the property companies undertake this exercise in considering their business expansion. Since the property incomes and property prices are strongly cyclical, the company is concerned to hedge the volatility of income during the economic downturn. The company will lose income on existing cash-flow but gain from the lower cost of expansion. The application of natural hedging is more significant for companies that relate their business towards the volatility of inflation and interest rates. Thus, this study focuses on the application of natural hedging in the insurers and annuity providers.

Consider a portfolio of life contingent liabilities consisting of term life insurance policies written on lives age 35 and immediate term life annuities written on lives age 65. The ultimate outcome focuses on the insurer's total liability should the mortality improve or deteriorate. If the mortality improves, the insurer will have loss on the annuity business and gain on the life insurance business. And if mortality declines, the effects are transposed. This study shows the effect on the insurer's liabilities if mortality risk increases or decreases as a result of a common shock. A good shock refers to mortality improvement, while a bad shock refers to mortality deterioration. In this subchapter, this study sets the shocks to be at $\pm$(5%, 10%, 25% and 50%). Here are the assumptions:

1. Countries observed are Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain.

2. Term insurance for (35) and annuity for (65) are based on the 1960–2007 period using the OL-GDP Model. Exceptions are Germany and Greece which start from 1970 and 1981 respectively.

3. For illustration purposes, the initial amount of the annual present value of a 60-year term insurance on (35) is set at £100 for all countries.

4. Concurrently, the annual present value of a 30-year temporary annuity derived from simultaneous equations is equivalent to £8,175, £7,979, £7,977, £7,961, £8,043, £8,880, £8,610, £7,489 and £8,255 for Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain respectively[1] and is payable at the beginning of each year, where the first payment is due at age 65.

5. Premium payments and annuity benefits are paid annually. Death benefits are paid at the end of the year of death; death must occur within the n years and the annuity is payable at the beginning of each year, as long as the annuitant is alive or upon reaching age 95; whichever is earlier.

[1] By doing so, the liabilities of present values for both insurance benefits and annuity payments arrived at about equal amount of £1,152.

6. The mortality shock, $\in$ is expressed as a percentage of the force of mortality μ_{x+t}, ranges from -1 to 1, that is, -1 $\leq$ $\in$ ≤ 1 with probability equal to 1. Without the shock, the survival probability for a life age (x) at year t is $p_{x+t} = \exp(-\mu_{x+t})$. With the shock, the new survival probability $\acute{p}_{x+t}$ is expressed as:

$$\acute{p}_{x+t} = (e^{-\mu_{x+t}})^{1-\epsilon} = (p_{x+t})^{1-\epsilon}$$

   If the shock lies between $0 < \epsilon \leq 1$, mortality experience improves. On the other hand, if the shock lies between -1 $\leq \epsilon < 0$, mortality experience deteriorates.

7. The term structure for insurance is 60 years, issued to (35) and the term structure for annuity is 30 years, issued to (65) respectively; and the effective interest rate is flat; a single interest rate is used at $i = 0.05$ for both insurance and annuity.

4.2.1 Expected Life Insurance Liabilities

For the term insurance, the present value of £1 paid at the end of the year of death is υ^k and the expected present value is:

$$A^1_{x:\overline{n}|} = \sum_{k=0}^{n-1} \upsilon^k {}_k p_x q_{x+k}, \tag{4.1}$$

where x is the age when the policy is issued ($x = 35$ in the analysis). For a benefit of F the expected present value is $FA^1_{x:\overline{n}|}$.

The present value of £1 per year, paid at the beginning of the year until the year of death or upon maturity, is:

$$\ddot{a}_{\overline{K(x:\overline{n}|)+1}|} = \frac{1 - \upsilon^{K(x:\overline{n}|)+1}}{d} \tag{4.2}$$

The expected present value is:

$$\ddot{a}_{x:\overline{n}|} = E\left[\ddot{a}_{\overline{K(x:\overline{n}|)+1}|}\right] = \sum_{k=0}^{n-1} \upsilon^k {}_k p_x \tag{4.3}$$

The net annual premium rate for £1 of benefit is determined so that the present value of net premiums is equal to the present value of benefits. This means:

$$P_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} \tag{4.4}$$

and for a benefit of F the annual premium is:

$$FP_{x:\overline{n}|} = \frac{FA_{x:\overline{n}|}}{\ddot{a}_{x:\overline{n}|}} \tag{4.5}$$

The insurer's loss, L, is defined as the random variable of the present value of benefits to be paid by the insurer less the annuity of premiums to be paid by the insured (see (Bowers et al., 1997)). This relationship is known as Equivalence Principle; under this principle, the expected loss is equal to zero;

$$E[L] = 0 \tag{4.6}$$

This is then where the expected present value of future benefits equals the expected present value of future premiums, E[PVFB] = E[PVFP]. In other words, at issue, this is equal to:

$$APV(Future\ Premiums) = APV(Future\ Benefits) \tag{4.7}$$

The contract of insurance is an agreement between the insurer and the insured. The insurer agrees to pay for insurance benefits while in exchange, the insured needs to pay the insurance premiums. Therefore, the insurer's net random future loss is defined by:

$$L = PVFB - PVFP \tag{4.8}$$

where PVFB is the present value at time of issue, of future premiums to be paid by the insured. For any paid premiums, P, the present value of the insurer's net loss if death occurs at time t is:

$$l(k) = v^k - Pa_{\overline{k}|} \tag{4.9}$$

Hence, the loss random variable, L corresponding to the loss function $l(k)$ is:

$$L = l(K) = v^K - Pa_{\overline{K}|} \tag{4.10}$$

In other words, if the insured dies at $K(x : \overline{n}|) = t$, then the insurer's net loss is the present value of the payment, less the present value premiums. For a £1 benefit, the loss is:

$$L = \upsilon^{K(x:\overline{n}|)+1} - P_{x:\overline{n}|}\ddot{a}_{\overline{K(x:\overline{n}|)+1}|} = \upsilon^{K(x:\overline{n}|)+1} - P_{x:\overline{n}|}\frac{1-\upsilon^{K(x:\overline{n}|)+1}}{d} \tag{4.11}$$

It follows from the definition of the net premium $P_{x:\overline{n}|}$ that the expected loss is zero. For a benefit of F, the loss is FL. In some cases, the loss can be negative in which, this turned out to benefit the insurer. However, on average, the loss is zero.

4.2.2 Expected Annuities Liabilities

For an annuitant (y), the present value of £1 per year paid at the beginning of the year is

$$\ddot{a}_{\overline{K(y:\overline{n}|)+1}|} = \frac{1-\upsilon^{K(y:\overline{n}|)+1}}{d} \tag{4.12}$$

The expected present value

$$\ddot{a}_{y:\overline{n}|} = E\left[\ddot{a}_{\overline{K(y:\overline{n}|)+1}|}\right] = \sum_{k=0}^{n-1} \upsilon^k\, {}_kp_y \tag{4.13}$$

The policy is purchased with a single payment of $\ddot{a}_{y:\overline{n}|}$ for n term. In our example y = 65 and the mortality table is based on annuity experience. For an annual benefit of b, the net single premium is $b\ddot{a}_{y:\overline{n}|}$. The company's loss per unit of benefit is

$$\ddot{a}_{\overline{K(y:\overline{n}|)+1}|} - \ddot{a}_{y:\overline{n}|} = \frac{1}{d} - \ddot{a}_{y:\overline{n}|} - \frac{\upsilon^{K(y:\overline{n}|)+1}}{d} \tag{4.14}$$

The expected loss is zero.

4.2.3 Liabilities of Term Life Insurance Portfolio

The portfolio has a term life insurance liability to pay a benefit of F at the end of the year of the death of (x) before reaching age 95; and a liability to pay a benefit of b at the beginning of each year as long as (y) is alive or upon reaching age 95; whichever is earlier. The total liability is

$$Fv^{K(x:\overline{n}|)+1} + b\ddot{a}_{\overline{K(y:\overline{n}|)+1}|} \tag{4.15}$$

To offset the liability the company has

$$FP_x\ddot{a}_{\overline{K(x)+1}|} + b\ddot{a}_y \tag{4.16}$$

The difference is the total loss:

$$L = Fv^{K(x)+1} + b\ddot{a}_{\overline{K(x)+1}|} - FP_x\ddot{a}_{\overline{K(x)+1}|} + b\ddot{a}_y \tag{4.17}$$

The expected loss is zero. However, this expectation is calculated under the assumption that mortality follows the life tables assumed in setting the premiums. In the next subsection, impacts on the insurance and annuity present values are presented with respect to mortality improvement or deterioration at certain level of shock ranges between ±(5–50%).

4.2.4 Effects of Changes in Mortality to Life and Annuity Payments

Tables 4.1 - 4.9 present the results of the present value of life insurance outflows and annuity outflows at time $t = 0$ for all countries under study, separately and in aggregate. They present the percentage deviation of the present value of benefits from the life insurance premiums and that of annuity payments from the total annuity premium collected at time $t = 0$. This study also shows the present value of the sum of both life insurance and annuity payments and the percentage of deviation from the present value of total premiums collected. Each result includes a shock improvement (Panel A) or shock deterioration (Panel B) relative to the mortality table, modelled by multiplying the force of mortality by a factor $1 - \epsilon$ in each year.

As presented in Table 4.1 for Austria, with a small mortality improvement shock, $\epsilon = 0.10$, the present value of the total annuity payments increases from £1,152 without shock to £1,170. In this scenario, annuity insurers will lose 1.57% [= (1,170 – 1,152) / 1,152] of their expected total payments. In this scenario, life insurers will gain 5.49% of their expected total payments. If the above life insurance and annuity are written by the same insurer, the shock has a much smaller effect on its business (a 1.96% gain).

Whereas, at the mortality bad shock of ϵ = –0.10, annuity insurers will gain 3.68% [= (1,152 – 1,110) / 1,110] of their expected total payments. In this scenario, life insurers will lose 5.06% of their expected total payments. If the above life insurance and annuity are sold by the same insurer, a bad shock has little effect on its business (a 0.69% loss). In a big good shock of 50%, $\epsilon = 0.50$, the present value of total annuity payments will increase by 14.89% and the life insurer will gain 33.94% of their total expected payments on average. The overall effects will be 9.53% gain on a big good shock. Writing both life and annuity business, reduces the impact of a big bad shock ϵ = –0.50 to a 4.99% loss.

Interestingly for Belgium, the present value of liability outflows for annuity registers a marginal gain of 0.56% although the mortality rate has improved by a small shock 5% ($\epsilon = 0.05$); see Table 4.2 for details. Hence, on aggregate, the insurer experiences a gain of 1.61% from both life and annuity outflows following a small shock mortality improvement of 5%.

On the other hand, a decrease of mortality rates by ϵ = -0.05 and -0.10 generate an aggregate gain of 0.31% and a loss of 0.30% respectively. Having said that, a major bad shock in mortality rates by ϵ = -0.50 has dragged the aggregate cash flow to 4.55% loss to the insurer. Comparing between the improvement and deterioration of mortality rate, the former shows a greater impact to the insurer.

In France, as expected, a small shock of improvement of ϵ = 0.05 and 0.10, results in a great impact on both life and annuity's present values. In addition, the insurer experiences almost a natural hedging (near zero) in a small shock of ϵ = 0.05, where the aggregate insurer's outflows for both life and annuity are almost balanced (-0.03%). Again, the insurer is still experiencing a natural hedging (less than a gain of 1%) when the shock improved by $\epsilon = 0.10$. Notwithstanding, a big shock ($\epsilon = 0.50$) gives the insurer a gain of 35.15% for life portfolio as compared to a loss of 16.81% for annuity portfolio. Therefore, on aggregate, the insurer is gaining 9.17% when the mortality rate registers a bigger improvement.

Austria

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,121	-2.69	1,154	0.18	2,276	-1.25
ϵ=0.10	1,089	-5.49	1,170	1.57	2,259	-1.96
ϵ=0.25	982	-14.74	1,222	6.06	2,205	-4.34
ϵ=0.50	761	-33.94	1,324	14.89	2,085	-9.53

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.58	1,124	-2.44	2,306	0.07
ϵ= -0.10	1,211	5.06	1,110	-3.68	2,321	0.69
ϵ= -0.25	1,291	12.00	1,070	-7.15	2,360	2.42
ϵ= -0.50	1,408	22.23	1,011	-12.24	2,420	4.99

Table 4.1: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Austria

When the mortality rate deteriorates from $\epsilon = 0.50$ to $\epsilon = 0.05$, the present value of life is increased from £1,184 to £1,426, bearing the loss to the insurer increases by ten-fold from 2.73% to 23.72% (see Table 4.3). However, if the insurer also underwrites the annuity portfolio, the loss can simply be reduced to 7.18%.

Similar experience is witnessed in Belgium, a small improvement by 5% ($\epsilon = 0.05$) on mortality rate in Germany (see Table 4.4) provides gains for both life and annuity's present values. The gains are registered at £30 (reduced insurer's liabilities from £1,152 to £1,122) and £13 (£1,152 to £1,139) for life and annuity outflows respectively. Accordingly, a small mortality improvement of 5% produces an aggregate gain of 1.93% to the insurer. Important highlight is captured when the

Belgium

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,122	-2.66	1,146	-0.56	2,268	-1.61
ϵ=0.10	1,090	-5.44	1,162	0.83	2,252	-2.30
ϵ=0.25	984	-14.59	1,214	5.34	2,198	-4.63
ϵ=0.50	765	-33.61	1,317	14.26	2,082	-9.68

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.55	1,116	-3.18	2,297	-0.31
ϵ= -0.10	1,210	5.01	1,101	-4.42	2,311	0.30
ϵ= -0.25	1,289	11.89	1,061	-7.89	2,351	2.00
ϵ= -0.50	1,407	22.06	1,003	-12.95	2,410	4.55

Table 4.2: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Belgium

mortality rate deteriorates even up to ϵ = -0.10; the insurer's aggregate outflows between life and annuity are almost balanced (almost zero). However, the life and annuity outflows start contradicting when mortality rate decreases by ϵ = -0.25 and ϵ = -0.50. The latter results in an aggregate loss of £94 [=£2,399 - £2,305] or 4.11% to the insurer.

Table 4.5 below shows the impact of mortality changes to the present value of life benefit and annuity payment for Greece. The small change of mortality rate by ϵ = 0.05 has reduced the present value of life benefits by £31 [=£1,152 - £1,121] or -2.71% which means a gain to the insurer.

France

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,120	-2.83	1,184	2.77	2,304	-0.03
ϵ=0.10	1,086	-5.78	1,200	4.11	2,285	-0.84
ϵ=0.25	974	-15.44	1,249	8.42	2,224	-3.51
ϵ=0.50	747	-35.15	1,346	16.81	2,093	-9.17

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,184	2.73	1,155	0.24	2,339	1.48
ϵ= -0.10	1,214	5.36	1,141	-0.97	2,355	2.20
ϵ= -0.25	1,299	12.75	1,102	-4.36	2,401	4.20
ϵ= -0.50	1,426	23.72	1,045	-9.35	2,470	7.18

Table 4.3: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of France

On aggregate the insurer loses 0.36%, 1.00%, 2.80% and 5.49% if the mortality rates deteriorated by ϵ = -0.05, -0.10, -0.25 and -0.50 respectively. Separately, a small change of ϵ = -0.05 gives loss of £30 [=£1,182 - £1,152] for life benefits and an aggregate gain of £8 [=£2,313 - £2,305] to the insurer, if the insurer writes both portfolio of life and annuity.

In Table 4.6, the results illustrate a similar pattern with earlier discussed countries like Austria and France, where an improvement of mortality rate has greater impact as compared to deterioration of mortality regardless the magnitude of the shock or ϵ. The insurer may not feel the impact even when the mortality rates improved by up to 10%. However, at 5% mortality deterioration, the insurer may

Germany

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,122	-2.66	1,139	-1.20	2,260	-1.93
ϵ=0.10	1,090	-5.43	1,155	0.21	2,244	-2.61
ϵ=0.25	984	-14.58	1,208	4.80	2,192	-4.89
ϵ=0.50	765	-33.62	1,312	13.86	2,077	-9.88

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.55	1,108	-3.86	2,290	-0.65
ϵ= -0.10	1,210	5.01	1,093	-5.12	2,303	-0.06
ϵ= -0.25	1,289	11.87	1,053	-8.64	2,342	1.61
ϵ= -0.50	1,406	21.99	994	-13.78	2,399	4.11

Table 4.4: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Germany

realise that the aggregate outflows will be experiencing a loss of £28 [=£2,333 - £2,305].

One the other hand, it is interesting to note that, a decrease by 5% (ϵ = -0.05) in mortality rates for Italy will not give any impact to the annuity's outflow as its present value is closed to zero. Nevertheless, the insurer will lose about £14 [=£1,152 - £1,138] when the ϵ = -0.10 for annuity portfolio.

Like Belgium and Germany, liability outflows for the Netherlands generate gains for both life and annuity portfolio given ϵ = 0.05, a small shock of mortality improvement. The same for the aggregate outflows where insurer enjoys an excess

Greece

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,121	-2.71	1,160	0.70	2,281	-1.01
ϵ=0.10	1,089	-5.53	1,176	2.06	2,265	-1.74
ϵ=0.25	981	-14.83	1,227	6.48	2,209	-4.17
ϵ=0.50	760	-34.05	1,327	15.19	2,087	-9.43

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.60	1,131	-1.88	2,313	0.36
ϵ= -0.10	1,211	5.11	1,117	-3.10	2,328	1.00
ϵ= -0.25	1,292	12.13	1,077	-6.52	2,369	2.80
ϵ= -0.50	1,412	22.52	1,019	-11.54	2,431	5.49

Table 4.5: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Greece

of £34 [=£2,305 - £2,271] or 1.45%. The same scenario of aggregate outflows (almost zero) when mortality rate deteriorates by a small shock (ϵ = -0.05).

Among all countries, changes of mortality rate have less impact on the Netherlands for all rates of change or ϵ as shown in Table 4.7. The most significant gain for life and annuity are 32.60% and 12.50% respectively. Whereas, the maximum losses are -20.64% for life benefits and -14.25% for annuity.

For Portugal, the insurer registers a positive outflow for both life and annuity when ϵ = 0.05 and 0.10; the aggregate present value for both are also increased by £55 [=£2,305 - £2,250] and £71 [=£2,305 - £2,234]. Table 4.8 also shows a gain of 1.06% and 0.43% when ϵ reduced by 5% and 10% respectively.

Italy

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,122	-2.64	1,180	2.41	2,302	-0.12
ϵ=0.10	1,090	-5.40	1,195	3.73	2,285	-0.84
ϵ=0.25	985	-14.54	1,245	8.00	2,229	-3.27
ϵ=0.50	764	-33.67	1,341	16.38	2,105	-8.64

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,181	2.53	1,151	-0.09	2,333	1.22
ϵ= -0.10	1,209	4.96	1,138	-1.28	2,347	1.84
ϵ= -0.25	1,287	11.72	1,099	-4.60	2,387	3.56
ϵ= -0.50	1,402	21.64	1,043	-9.48	2,445	6.08

Table 4.6: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Italy

Similar to Belgium, Germany and the Netherlands, an improvement of ϵ = 0.05, also provides a gain to the annuity portfolio. Moreover, an improvement of ϵ = 0.10 also generated a gain of £7 on the annuity's present value. In addition, among all countries, Portugal registers a maximum aggregate gain of 10.37% to the insurer's outflow for a major good shock of ϵ = 0.50.

As for Spain, (please refer to Table 4.9), a major good shock of (ϵ = 0.50) has resulted in a maximum gain of 34.13% for life and a maximum loss of 15.67% for annuity portfolio as compared to other countries under study. Interestingly, a small shock of either ϵ = 0.05 or -0.05 of mortality, generates almost balanced aggregate present values (almost zero) for both life and annuity portfolios.

Netherlands

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,123	-2.53	1,148	-0.38	2,271	-1.45
ϵ=0.10	1,093	-5.18	1,164	0.98	2,256	-2.10
ϵ=0.25	991	-13.97	1,215	5.42	2,206	-4.27
ϵ=0.50	777	-32.60	1,316	14.25	2,093	-9.17

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,180	2.42	1,118	-2.95	2,299	-0.27
ϵ= -0.10	1,207	4.74	1,104	-4.16	2,311	0.29
ϵ= -0.25	1,281	11.19	1,065	-7.56	2,347	1.82
ϵ= -0.50	1,390	20.64	1,008	-12.50	2,398	4.07

Table 4.7: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Netherlands

To summarise, mortality improvement has greater impact (huge gain for life's outflow and severe loss for annuity's outflow) as compared to mortality deterioration for all countries (Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain). Moreover, given the reality of longer life expectancy, serious attention by insurers is required in managing longevity risk. This can be done using natural hedging (Luciano et al., 2012) and capital markets (Kim and Choi, 2011).

Natural hedging aims to stabilise the aggregate outflows of the company. This is done through the interactions between life and annuity portfolio with respect to a change in mortality (improvement or deterioration). Under these circumstances,

Portugal

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,121	-2.71	1,129	-2.04	2,250	-2.38
ϵ=0.10	1,089	-5.54	1,145	-0.62	2,234	-3.08
ϵ=0.25	982	-14.81	1,199	4.01	2,180	-5.40
ϵ=0.50	761	-33.95	1,304	13.21	2,066	-10.37

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.61	1,098	-4.72	2,280	-1.06
ϵ= -0.10	1,211	5.12	1,083	-5.99	2,295	-0.43
ϵ= -0.25	1,293	12.18	1,043	-9.52	2,335	1.33
ϵ= -0.50	1,414	22.68	983	-14.66	2,397	4.01

Table 4.8: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Portugal

the insurer is expected to have a positive outflow for life benefits and negative outflow for annuity payments during mortality improvement. And vice versa when mortality rates are deteriorating. Based on that expectation, only Austria, France, Greece, Italy and Spain experience the expected patterns for both life and annuity portfolio as well as on an aggregate basis.

On the other hand, Belgium, Germany, the Netherlands and Portugal interestingly record a positive impact for both life and annuity at $\epsilon = 0.05$. Further observations on mortality rates for all countries show that the truncation during ages 87–89 has affected both life and annuity liabilities for countries like Belgium, Germany, France, the Netherlands and Portugal (see Appendix II for details). Hence,

Spain

Panel A: Improvement level aged (35) / Improvement level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ=0.05	1,121	-2.71	1,167	1.31	2,288	-0.70
ϵ=0.10	1,089	-5.54	1,183	2.66	2,272	-1.44
ϵ=0.25	981	-14.85	1,234	7.05	2,215	-3.90
ϵ=0.50	759	-34.13	1,333	15.67	2,092	-9.23

Panel B: Deterioration level aged (35) / Deterioration level aged (65) = 1

	Present Value Life Benefits Payments, £	△ Life, %	Present Value Annuity Payments, £	△ Annuity, %	Total Present Value, £	△ Total, %
ϵ=0	1,152	-	1,152	-	2,305	-
ϵ= -0.05	1,182	2.60	1,138	-1.26	2,320	0.67
ϵ= -0.10	1,211	5.11	1,124	-2.48	2,335	1.32
ϵ= -0.25	1,292	12.12	1,084	-5.89	2,376	3.11
ϵ= -0.50	1,411	22.47	1,027	-10.90	2,438	5.78

Table 4.9: Results for 5%, 10%, 25% and 50% Mortality Improvement or Deterioration Relative to Life and Annuity Mortality Tables (the present values are in thousands) of Spain

improvements in mortality rates enforce serious concerns to insurers not only for annuity liabilities but on both life and annuity liabilities for some countries when mortality rate improves. A securitisation of longevity risk through capital market solutions can be another approach towards risk management.

4.3 Economics of Mortality Risk and Securitisation

This subsection describes the urgency to attend to the longevity risk. This phenomenon has alarmed annuity providers as to how they manage the longevity risk efficiently. Historically, life securitisation transactions began in the late twentieth century (end-1990/early-2000). Although they can be considered as relatively as new, these insurance-related securities have become famous after the global financial crisis in 2008. Securitisations can be classified into two categories; securities backed by an asset like corporate bonds or mortgages (asset-backed securities) and securities not backed by an asset like futures, options and swaps (non-asset-backed securities).

4.3.1 Changes in Mortality Experience

Over the past few decades, the world has been experiencing an unprecedented increase in life expectancy globally (Macdonald et al., 1998). This phenomenon has benefited the individuals, but at the same time has threatened the annuity providers. Annuity providers often look for effective tools and measures in addressing this issue. Viably, they may consider using financial markets to mitigate the longevity risk by securitising parts of their portfolio. Securitisation acts as a hedge for the insurer's portfolio by transferring the risks of the portfolio to third parties and serves as a substitute for reinsurance (Lorson and Wagner, 2014).

Unlike the non-life segments, where securitisation transactions like catastrophic bonds were introduced long time ago, the life securitisation market is still at an infancy at age (Deutsche Bank, 2010). To draw on history, the first mortality-linked securitisation transaction was carried out by the world's second largest reinsurer, Swiss Re in 2003. The issue of the Vita Capital bond reduced Swiss Re's exposure on longevity risk. The principal payments of this bond were based on a predefined mortality index (Blake et al., 2006). Another attempt of longevity bond issuance was made by the European Investment Bank in 2004. The bond with coupon payment depending on the survival of English and Welsh males aged 65 years has a contract duration of 25 years. However, this £540 million worth volume did not attract enough investors and was abandoned in 2005 (Chen and Cummins, 2010). After these initial attempts, the life securitisation market has flooded the industry with increased volumes and transactions until 2007. During the recent financial crisis, the issuing of life securitisations dropped and now is still in a recovery phase.

Numerous studies have been conducted earlier on hedging the longevity risk. Wang et al. (2011) applied the concept of reverse mortgages to hedge longevity risk for life insurance companies. Alternatively, various other methods of hedging longevity risk have been demonstrated by many researchers. Among others, Delta-Gamma hedging (Luciano et al., 2012), natural hedging (Gatzert and Wesker, 2012; Wang et al., 2010) and annuity securitisation (Kim and Choi, 2011). Utilising the percentile tranching approach, Kim and Choi (2011) applied the concept of an inverse survivor bond to a fictional portfolio of Australian annuity contracts. They assumed that by investing in this bond, investors can achieve their targeted yield.

This subchapter aims to review and analyse the previous studies related to hedging longevity risk. Main discussion of this subchapter would be focused on the study conducted by Kim and Choi (2011) and Lorson and Wagner (2014). They applied a percentile tranching method in their study. The chosen percentiles are then linked directly to Standard & Poor's (S&P)[2] rating classes, so that the quality of the securitisation tranche become transparent to the investors. Besides that, the issuer also has an opportunity to approach specific investors based on their risk-appetite towards the securitisation offer.

Subsequently, for future research, this study aims to extend the methods of hedging the longevity risk on the annuity portfolio. This can be done through annuity securitisation. Motivated by Lorson and Wagner (2014), this study will employ a similar concept. However, this study utilises the OL-GDP mortality model and focuses on nine selected Eurozone countries namely Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain. The calculation of the bond price is done in a three-step process. First step is the calculation of future mortality rates using the OL-GDP model. In the second step, the forecasted annuity portfolio is segregated into different tranches with the help of the percentile tranching method as per Kim and Choi (2011). Hence, attachment and

[2]Standard & Poor's Global Ratings is an American financial services company that has been operating for more than 150 years. They are the world's leading provider of credit ratings. They educate the market participants in making investment decisions confidently.

detachment points are applied on the individual tranches based on the S&P ratings for insurance-linked securities. Finally, based on classical bond pricing, the pricing of each tranche of the annuity securitisation is computed. The principal payments that investors receive depend on the survival distribution of the underlying portfolio randomly. The higher the number of actual survivors compared at to expectations, the lower the amount of the principal payment.

4.3.2 Longevity Bonds

Longevity bonds are used by insurance companies to hedge their annuity portfolio against mortality changes. The structure of this bond is illustrated in Figure 4.1.

In the middle of the bond structure stands a special-purpose vehicle (SPV). This legal entity acts as an inter-mediator that brings together the issuer (the insurance company) and the investors who want to engage in the annuity portfolio. From the insurer's viewpoint, he is paying out the annuities to its annuitants, while at the same time transferring the premiums received from the annuity sales to the SPV. Meanwhile, from the investor's viewpoint, through the investment investors make into the bonds, they receive the coupon payments and principal from the SPV. Being the intermediary, SPV provides contingent payments to the insurance company (securing thus the annuity payments) and issues a survivor bond which pays regular coupon payments to the investors. Normally, the principal is paid as a lump sum at the maturity of the bond, while the coupon payment is paid annually and subject to the survival rate of the underlying portfolio, and thus it is exposed to risk. In this study, the principal payment will be at risk, whereas the coupon payments are secured.

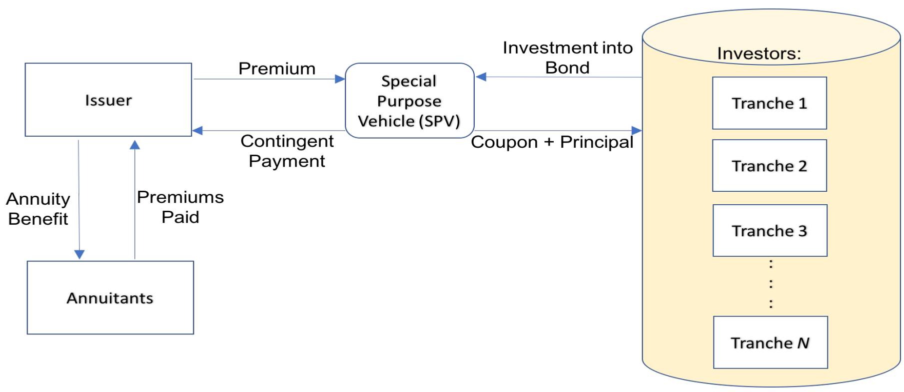


FIGURE 4.1: Structure of a Longevity Bond

In attracting more investors, the longevity bond is divided into several different tranches. Each number of tranche carries different weights of risk. The investors in the first tranche are the first to lose their coupon or principal payments if the number of actual survivors in the portfolio is higher than expected. Next, the loss goes to the second tranche if the first tranche is exceeded, and the process goes on. Different tranches pose different characteristics of risk and different prices for the bond.

4.4 Securitisation Framework for Annuity Portfolio

This section describes the proposed bond structure as a way to securitise a life annuity portfolio. This may help insurers to face longevity risk within their annuity portfolios. The process of designing the bond must be well structured, scrutinised and prudent. Fundamentally, the process of pricing the bond involves:

1. Forecasting future mortality rates
2. Tranching process and excess loss
3. Pricing of bond

4.4.1 Forecasting Mortality Rates with OL-GDP Model

The evolution of mortality models was pioneered by Lee and Carter (1992). Since then, a wide range of stochastic models for forecasting mortality rates have been developed extensively in the last decade. Although, the Lee-Carter model is still being the most popular model and widely used in the literature for valuing life portfolios and as a basis for insurance securitisation (Denuit et al., 2007; Kim and Choi, 2011), this study applies the improved mortality model with GDP effect, OL-GDP as presented in Chapter 2.

Applying the OL-GDP model requires a two-step process. First, the model parameters in Equation 2.7 are estimated based on the observed mortality rates and, second, the projections for the future are performed.

The time-dependent factors of OL-GDP are forecasted using the Arima function from R package forecast, "forecast.Arima".

4.4.2 Tranching Process and Excess Loss

Many studies conducted earlier (Wang, 2000; Wills and Sherris, 2010; Kim and Choi, 2011; Lorson and Wagner, 2014), applied the tranching approach in pricing the bond. The definition of the individual tranches to be securitised varies. For instance, Kim and Choi (2011) defined the attachment point of the first tranche at the median survival probability, whilst Lorson and Wagner (2014) linked the attachment point to the S&P default table for insurance-linked securities[3] (please see Appendix 1). This subsection undertakes the definition of individual tranches to be securitised and the calculation of the excess loss for each tranche. Inspired by Lorson and Wagner (2014), this study adopts a similar approach in defining the tranche by considering N as different tranches in the proposed longevity bond. The attachment points $p_{x,t}^{(j-1)}$ and detachment points $p_{x,t}^{(j)}$ for tranche j, where $j = 1, 2, \cdots, N$, are defined as a percentile of the cumulative forecasted survival distribution. In this approach, the detachment point of the last tranche is defined as the 100^{th} percentile of the survival distribution, i.e. $p_{x,t}^{(N)} = 1(100$ percentile). This study assumes that the attachment and detachment points for the remaining tranches are flexible and defined according to the intended tranche composition of the securitisation. The percentile below the attachment point of the first tranche $p_x^{(0)}$ is the part retained by the issuer. This can be considered as a first loss position. Subsequently, the percentile within the attachment and detachment points will be borne by the investors in Tranche j.

In calculating the excess loss for each tranche, this study assumes the insurer pays £1 to each annuitant. Consider the attachment points $p_{x,t}^{(j-1)}$ and the detachment points $p_{x,t}^{(j)}$ for each tranche j, where $j = 1, 2, \cdots, N$ are given, and let l_x represent the initial population of annuitants of age x at the beginning of the securitisation ($t = 0$); thus, the actual number of survivors in each group (tranche) is a random variable of survival distribution rates, $p_{x,t}$, where $l_{x+t} = l_x \cdot p_{x,t}$, describes the actual number of survivors at age $(x + t)$. As described in Figure 4.2, each dot

[3] S&P provides cumulative default probabilities for different rating classes and different maturities. In Appendix I, this study reports the S&P cumulative default probabilities for insurance-linked securitisations corresponding to the rating classes AAA to B– and maturities ranging from $T = 1$ to 30 years.

represents one realisation of the life expectancy simulation for individual ages x at a given time t after securitisation. The dashed lines indicate the attachment and detachment points of the observed tranche j (at time t). If a survival realisation lies within the compound of the loss (between the dashed lines), then, the tranche j has to bear the loss. The loss for tranche j is triggered by the grey shade. Thus, the random variable of the excess loss for the jth tranche for individuals aged x at the time of securitisation and t years after securitisation, can be described by:

$$L_{x+t}^{(j)} = [L_{x+t} - L_{x+t}^{(j-1)}]^{+} - [L_{x+t} - L_{x+t}^{(j)}]^{+} \tag{4.18}$$

where, $[\ *\ ]^{+}$ stands for max $(0,*)$.

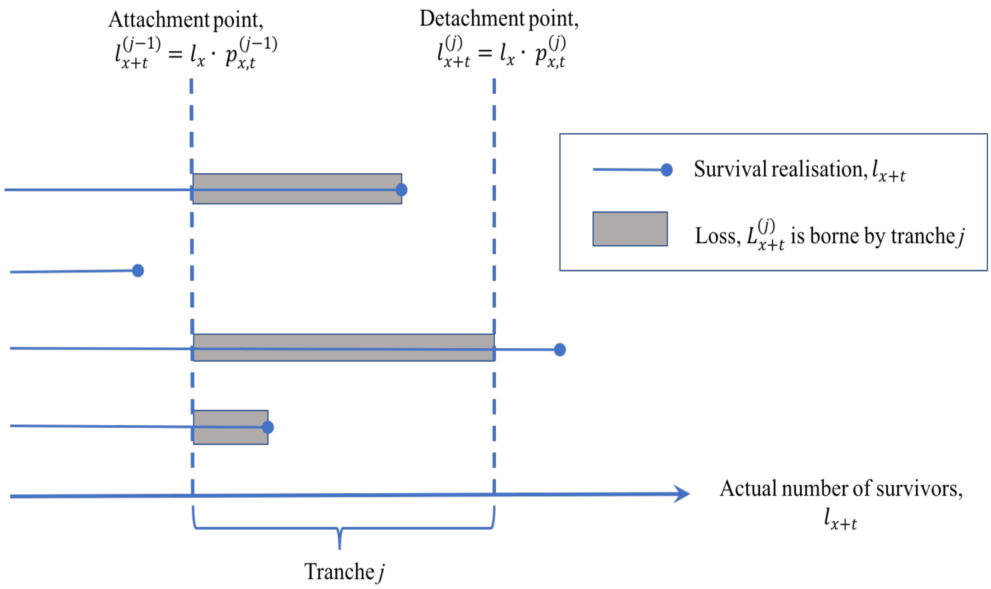


FIGURE 4.2: Illustration of the Excess Loss $L_{x+t}^{(j)}$ Calculation [see Equation 4.18] for Tranche j at Time t for Annuitants Aged $x+t$ (age x at the time of securitisation)

4.4.3 Pricing of Longevity Bond

Having determined the excess loss that may be incurred during the tranching process, the next step is to price the bond appropriately. In this practice, if the number of survivors is higher than expected, the tranche will be triggered; then the principal payment of this tranche is reduced proportionally. The proportional default factor for each tranche j and time t is defined by:

$$\Lambda_{x+t}^{(j)} = \frac{L_{x+t}^{(j)}}{l_{x+t}^{(j)} - l_{x+t}^{(j-1)}} \tag{4.19}$$

where, $0 \leq \Lambda_{x+t}^{(j)} \leq 1$.

The process involved in pricing the single tranche and the annuity securitisation of a longevity bond consists of two parts, the principal payment and the coupon payment.

The structure of the bond is designed so as to allow the back payment of the nominal FV to be distributed over the duration of the contract T with T being an equal size of payments which is defined by FV/T. Since the principal payment is linked to risk, the random variable of this is:

$$\mathcal{P}_{x+t}^{(j)} = (1 - \Lambda_{x+t}^{(j)}) \cdot \frac{FV}{T} \tag{4.20}$$

for the tranches $j = 1, 2, \cdots, N$ and the annuitants ages $x + t(t = 1, 2, \cdots, T)$. The principal value can vary between 0 (where the actual survival rate is greater than the detachment points of the jth tranche) and FV/T (a full payment of where the actual survival rate is lower than the corresponding attachment point of the jth tranche). In other words, $0 \leq \tilde{\mathcal{P}}_{x+t}^{(j)} \leq FV/T$.

On another part, the coupon payment is being paid annually based on the outstanding principal. Normally, the annual interest rate, $c^{(j)}$ applied to each tranche, j, comprises of a benchmark yield, y set by LIBOR or EURIBOR for instance, plus a tranche-specific spread, $s^{(j)}$;

$$c^{(j)} = y + s^{(j)} \tag{4.21}$$

As the calculation of the interest amount is defined by the outstanding amount of debt toward the investor;

$$D_t = FV - (t-1) \cdot FV/T = (T - t + 1) \cdot FV/T \tag{4.22}$$

Thus, the coupon payment is given by:

$$\mathcal{C}_t^{(j)} = D_t \cdot c^{(j)} = (T - t + 1) \cdot \frac{FV}{T} \cdot (y + s^{(j)}) \tag{4.23}$$

for all tranches $j = 1, 2, \cdots, N$ in times $t = 1, 2, \cdots, T$.

Finally, the price of the longevity bond is derived by adding the components of principal payment and coupon payment. The securities price $P_x^{(j)}$, in time $t = 0$ for individual age x (at the time of securitisation) and for tranches, $j = 1, 2, \cdots, N$, corresponds to the present value of the sum of principal payments, $P_x^{(j),\mathcal{P}}$ and the sum of all discounted coupon, $P_x^{(j),\mathcal{C}}$ payments until contract maturity, T. The present value of the bond price, $P_x^{(j)}$ is:

$$\begin{aligned}
P_x^{(j)} &= P_x^{(j),\mathcal{P}} + P_x^{(j),\mathcal{C}} \\
&= \sum_{t=1}^{T} E(\tilde{\mathcal{P}}_{x+t}^{(j)} \cdot (1 + r_f)^{-t} + \sum_{t=1}^{T} \mathcal{C}_t^{(j)} \cdot (1 + r_f)^{-1} \\
&= \sum_{t=1}^{T} = \frac{1}{(1 + r_f)^t} \cdot [E(\tilde{\mathcal{P}}_{x+t}^{(j)}) + \mathcal{C}_t^{(j)}] \\
&= \frac{FV}{T} \cdot \sum_{t=1}^{T} \cdot \frac{1}{(1 + r_f)^t} \cdot [E(1 - \tilde{\Lambda}_{x+t}^{(j)}) + (T - t + 1) \cdot (y + s^{(j)}]
\end{aligned} \tag{4.24}$$

with r_f denoting the risk-free interest rate and $E(*)$ denotes the expected value operator.

4.5 Conclusion

Longevity risk has long been an issue for insurance companies and also annuity and pension providers. There are numerous ways being used as alternative risk transfers. Among others are reinsurance, natural hedging or transferring the risk to the capital market through bond issuance, swaps and options. This study introduces a natural hedging to hedge the longevity risk. Natural hedging utilises the interaction between life insurance and annuities that observe the change of mortality as a way to stabilise the company's cash outflows. Through the empirical evidence, this study suggests that natural hedging is an important factor that contributes to the difference of the annuity price after managing other variables.

On another section, this study proposed a framework for annuity securitisation through the introduction of a longevity bond. Through the forecasted mortality rates and the tranching process, a longevity bond is then priced appropriately. This approach is in line with the government's concern on longevity risk. In addressing this issue, regulators have developed risk-based capital regimes that prompts the market players like insurers to consider the business mix which allow them to optimise the management and allocation of their capital. Under this regime, all market players can choose either to acquire annuity and longevity risk (by offering a niche longevity instrument like bond) or to reduce their exposure due to uncertainty of future mortality improvements by abiding to the minimum future capital requirements that has been set by the regulator at the same time.

Chapter 5

Conclusion

This study has shown that the growth of ageing population is evident as a positive milestone of society's well-being. Many factors contribute to human longevity, its macroeconomic factors affecting it the most. Having said that, each country experiences a different trend of longevity improvement. For instance, the life expectancy for European countries has risen significantly over the past few decades. This has led insurance and pension providers to revisit their ways of pricing their products and of hedging longevity risk. Hence, this study has addressed this concern by developing a robust mortality model. In achieving it, the new model has improved the existing O'Hare and Li (OL) model and has considered the influence of gross domestic product (GDP) into the new model. Therefore, the development of the new mortality model through the combination of the OL model and GDP-age dependent factor has increased the predictive power as compared to other mortality models.

Economic growth and health status are two major factors being deeply studied in gauging nations' social development and public policy. Due to these facts, socio-economic factors such as GDP, inflation and unemployment have clearly been observed to have a causal effect on mortality experience. The majority of related studies showed that improvements in mortality have been accompanied by growth in GDP. Hence, this study has found that the mortality rate and GDP growth are highly correlated in the countries sampled (Austria, Belgium,France, Germany, Greece, Italy, the Netherlands, Portugal and Spain). Moreover, the mortality of GDP relationship has been found to be more significant for older ages regardless of gender of which this age group is the most significant group that exposed timely to the longevity risk.

Following this, a new GDP-age dependent model has been introduced by assessing the role of economic growth on mortality dynamics. As shown in the equation below, the additional factor, $\mathbf{cg_x}$ is a vector of correlation coefficients between logarithmic of GDP per capita and mortality rates for ages 0–89:

$$\ln(m_{x,t}) = b_x^1 + k_t^1 + \mathbf{cg_x}(\bar{x} - x)k_t^2 + (\bar{x} - x)^+ k_t^3 + ([\bar{x} - x]^+)^2 k_t^4 + \gamma_{t-x} + \epsilon_{x,t}$$

where factor b_x^1 describes the average age-specific mortality, that ensures the basic shape of the mortality curve over ages is in line with historical observations; factor k_t^1 represents the changes in the mortality level; whilst k_t^2 factor allows changes in mortality to vary between ages reflecting the historical observations that improvement rates can differ for different age classes; k_t^3 and k_t^4 model the effects specific to the lower ages; γ_{t-x} models the cohort effect, $(\bar{x} - x)^+ = max(\bar{x} - x, 0)$; and $\bar{x}$ is the average of age considered.

This study validates the performance of the proposed model, OL-GDP against other mortality models, i.e. LC, OL and LC-GDP through several measures and approaches. Evidently, with the inclusion of the GDP growth indicator in the OL-GDP model, this study finds the best fitting model for most Eurozone countries, especially for France, Germany and Italy. Except for female, male and unisex produced the best fitting of OL-GDP compared to other models. This study notes the impact of GDP is significant especially for the most populous on sample countries. What is more, OL-GDP model also demonstrates better quality forecasts of mortality rates. Essentially, the forecasting shows significant improvement in forecasting results for shorter (5 years) and longer (15 years) periods of times in comparison to the LC model which acts as the reference model. The impact of economic growth on mortality dynamics has proved to be noteworthy.

Second, in validating the model's robustness and intuitiveness, this study has proved that the OL-GDP model responds well to the parameters variations compared to alternative models. Sensitivity analyses was conducted in assessing the impact of the model onto the parameters variations. The analysis includes the variations of the age range used, varieties of correlation coefficients, different age parameters and also the variations of the time-dependent factor of k_t^2. In addition, this study has shown that the model also responds well from a financial perspective. Organisations like life insurers and pension funds are sensitive towards the change of mortality rates. Drawing on the variations and range of modifiable factors in the model, this study observes that these variations have impacted the actuarial present values (APVs) of the actuarial products. In reference, this study has focused the research on the basic products of life insurance and annuity, namely annuity due, term insurance, pure endowment and endowment insurance. Moreover, this study has extended the investigation to actuarial reserves as well.

For better understanding, this study has conducted similar studies on other mortality models like Lee and Carter (LC), O'Hare and Li (OL) and Niu and Melenberg (LC-GDP). As mortality experience is unique across countries, different mortality models suit each country differently. However, in general, the OL-GDP model suits best most countries and scenarios. As the impact of mortality improvements is more significant to the insurers and pension fund providers, they need to be more mindful and vigilant in choosing reliable mortality models. A suitable choice of mortality models is deemed crucial. Further investigation in the future will certainly shed some light on longevity risk.

Third, this study has further investigated further the risk of mortality rate changes. The ultimate outcome focuses on the insurer's total liability should the mortality improve or deteriorate. If the mortality improves, the insurer will have a loss on the annuity business and a gain on the life insurance business. And if mortality declines, the effects are transposed. This study has shown the effect on the insurer's liabilities if the mortality risk increases or decreases as a result of a common shock. A good shock refers to mortality improvement, while a bad shock refers to mortality deterioration. Therefore, this study has illustrated that mortality improvement has greater impact (huge gain for life's cash flow and severe loss for annuity's cash flow) as compared to mortality deterioration for all countries (Austria, Belgium, France, Germany, Greece, Italy, the Netherlands, Portugal and Spain). Moreover, given the reality of longer life expectancy, serious attention by insurers is required when managing longevity risk. This can be done using natural hedging (Luciano et al., 2012) and capital markets (Kim and Choi, 2011).

Fourth, in reference to the concern of risk of mortality rate changes, this study has introduced a framework for an annuity securitisation with the help of longevity bond. At the same time, this study has taken the perspective of the annuity issuer and calculated the price of hedging for the company. The proposed framework applied a tranching approach for the securitisation based on the percentile tranching method and designed the bond in a way that the principal payments are risky, i.e. depend on the survival rates of the underlying portfolio of annuitants. To do so, the proposed framework first used the OL-GDP model for European countries and calculated estimates of future mortality rates. Next, the tranching process is done by segregating the portfolio into different rated tranches according to different risk profiles based on the S&P ratings for insurance-linked securities. Finally, the proposed framework determines the price of the longevity bond for hedging the contracts against longevity risk.

With that, this study foresees that, considering the importance of having a robust mortality model on forecasting, pricing, reserving and managing insurance liabilities, OL-GDP model can be an alternative model for insurers and pension providers in pricing their products and managing their cash flows efficiently. Therefore, OL-GDP model helps insurers to price life insurance better and allocate sufficient insurance reserve safe. Moreover, OL-GDP model helps examine mortality and longevity risks in a more prudent manner.

Although the new model is able to estimate the historical trends in mortality data efficiently, the estimation may not always be a sensible procedure to employ in the long-run and across the globe. Perhaps, as mentioned earlier, different countries experience different pace of economic growth, emerging and frontier markets tend to enjoy higher GDP growth rates on average, compared to their developed counterparts. Moreover, as the mortality in the Eurozone countries has not always declined along the path represented by the plot of k, this will reach to the impossibly high levels of mortality rates eventually. Thus, this will disrupt the accuracy of the mortality estimations. Perhaps the institutional bodies doing the demographics research may produce some insights that the twentieth-century trends will continue in the future. Therefore, further investigation on this will certainly shed some light on mortality studies. An in-depth study on this subject will absolutely give a clearer picture in explaining the mortality trends more precisely. At the same time, this will definitely improve the mortality forecasting accuracy.

There are rooms for extending this study's scope. Future studies may work on the OL-GDP model to explore various ways of managing mortality and longevity risks. This can be done by implementing the natural hedging strategy or structuring capital market solutions (securitisation and financial derivatives) for efficient liabilities outflow management or optimal securities/derivatives pricing respectively.

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Appendix

Appendix I

S&P default table for insurance-linked securitisations reporting cumulative default probabilities (in %) for different ratings and maturities (in years)

Maturity	AAA	AA+	AA	AA−	A+	A	A−	BBB+
1	0.003	0.010	0.015	0.025	0.040	0.060	0.085	0.234
2	0.027	0.048	0.074	0.106	0.150	0.200	0.264	0.514
3	0.052	0.085	0.133	0.188	0.260	0.340	0.443	0.850
4	0.076	0.123	0.191	0.269	0.370	0.480	0.621	1.246
5	0.100	0.160	0.250	0.350	0.480	0.620	0.800	1.704
6	0.122	0.192	0.310	0.397	0.531	0.655	0.966	1.805
7	0.144	0.224	0.420	0.543	0.719	0.887	1.287	2.261
8	0.204	0.311	0.549	0.713	0.937	1.152	1.648	2.756
9	0.276	0.414	0.700	0.909	1.184	1.451	2.047	3.284
10	0.362	0.536	0.872	1.130	1.458	1.782	2.479	3.842
15	1.037	1.447	2.078	2.617	3.237	3.864	5.051	6.936
20	2.175	2.893	3.858	4.690	5.586	6.493	8.068	10.279
25	3.804	4.853	6.133	7.223	8.337	9.454	11.284	13.667
30	5.885	7.241	8.781	10.066	11.329	12.580	14.553	17.003
$s(\ell)$	45	65	80	110	140	170	210	300

Maturity	BBB	BBB−	BB+	BB	BB−	B+	B	B−
1	0.353	0.547	1.632	2.525	3.518	4.510	5.824	8.138
2	0.825	1.279	3.211	4.946	6.915	8.885	11.751	16.674
3	1.405	2.177	4.758	7.230	10.095	12.960	17.152	24.004
4	2.073	3.213	6.276	9.380	13.037	16.694	21.921	30.025
5	2.812	4.359	7.763	11.403	15.745	20.087	26.089	34.945
6	2.980	6.316	8.327	12.175	16.832	21.462	27.947	38.234
7	3.672	7.434	9.598	13.826	18.895	24.083	30.999	41.476
8	4.390	8.529	10.831	15.387	20.800	26.457	33.680	44.209
9	5.127	9.598	12.025	16.862	22.563	28.610	36.046	46.543
10	5.876	10.637	13.179	18.258	24.197	30.565	38.145	48.559
15	9.684	15.418	18.383	24.234	30.849	38.096	45.822	55.592
20	13.414	19.591	22.777	28.944	35.737	43.198	50.706	59.851
25	16.980	23.300	26.570	32.808	39.556	46.958	54.169	62.789
30	20.367	26.665	29.933	36.108	42.709	49.936	56.845	65.022
$s(\ell)$	340	400	490	560	620	680	750	800

Note: For each rating, the corresponding (yearly) spread $s(\ell)$ is indicated in basis points (1 bp = 0.01%)
Source: S&P at www.standardandpoors.com

Appendix II

Mortality Rate based on OL-GDP Model for Austria, Belgium, France, Germany, Greece, Italy, Netherlands,Portugal and Spain